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The use of pseudo-stochastic effects in a tree growth projection system.

by

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Abstract

The stochastic method used by the CAlifornia Conifer Timber Output Simulator (CACTOS) for modifying growth predicitons is presented. This method is applied to DBH^2 and total height growth in order to capture the inherent variation in grow predicitons. A quintupling process is used which uses computer-generated random numbers that can be replicated. Hence, this process is called pseudo-stochastic.

The heteroscedasticity and skewed nature of the prediction errors are shown and incorporated into the pseudo-stochastic method. A description of the implementation of this technique into CACTOS is shown. Also, an example is provided to show the effect that pseudo-stochastics have on long term growth projections.

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INTRODUCTION

The objective of modelling tree growth is to predict the average growth response of an individual tree given its species, size and level of competition. The differences between the actual tree growth and the predicted tree growth is called the error in predictions and the variance of these errors is called the unexplained variation. To improve growth estimates, a modeller tries to minimize the unexplained variation without introducing bias into the estimates.

Incorporating tree growth models into computer programs that simulate the effects of alternatives stand management practices is now common **(Stage, 1973, Ek & Monserud, 1974, Krumland & Wensel, 1980, Wensel and Daugherty, 1985)**. CACTOS, the CAlifornia Conifer Timber Output Simulator **(Wensel & Daugherty, 1985)** uses the tree growth projection system described by **Wensel & Koehler(1985)**. The purpose of this paper is to describe the stochastics effects incorporated into CACTOS in order to represent the unexplained variation in predicting growth of tree diameter at breast height (DBH) and total height (HT).

While the purpose of modelling tree growth is to predict the average growth response of trees under diverse conditions, two trees that are the same size and under the same amount of measured competition will not necessarily grow at the same rate **(Bella, 1970)**. This may be caused by errors in the measurements or by factors not considered in the models. If these deviations from the average growth response are not incorporated into the simulator, the simulator may produce sluggish predictions of growth responses after thinnings and harvest. Additionally, the dispersion into diameter classes will be retarded if stochastic modifiers are not added to the growth equations **(Krumland,1982)**. Hence, using a stochastic representation of the errors in predictions is important in order to adequately predict changes in timber stands.[1/]

[1/] Note that this is not necessary for estimating mean effects as done in inventory updates.

METHODS

In CACTOS, the four tree variables that need to be predicted for each growth cycle are as follows:

DBH	diameter at breast height
HT	total tree height
CR	live crown ratio
TPA	number of trees per acre for each tree record

Stochastic variation is not added to the crown ratio estimates. However, crown ratio is a function of total height and stand density and therefore crown recession will indirectly have stochastic effects through the stochastic effects on height growth. For these reasons, and the lack of data for both the mortality and crown recession models, stochastic effects will be computed for only the DBH and HT growth models.

Since an independent data set does not exist for use in determining the unexplained variations, the stem analysis database that was used to develop the growth equations will also be used. A description of this data set is given by **Wensel & Koehler (1985)**. The design of the stem analysis samples includes the following elements. First, clusters of 2 or 3 plots were selected and, second, 5 to 10 trees were selected for felling on each plot. This allows for the potential estimation of the variation among clusters, plots, and trees, as well as differences between time periods.

Two questions that need to be answered are (1) what is the nature of the deviations from the average growth response and (2) how should these deviations be incorporated into CACTOS. The residuals from the diameter-squared and total height growth models are heteroscadastic with respect to predicted growth. That is, the error variance is proportional to the square of predicted growth. This heteroscedasticity can be seen by looking at the residual scatterplot shown for ponderosa pine in **Figure 1 (Wensel and Koehler, 1985)**. Hence, the following model is appropriate for expressing the growth functions for both DBH^2 and total height:

$$Y_{ijk} = f(X_{ijk}) \times (1 + E_{ijk}) \qquad [1]$$

where

Y_{ijk} = actual growth of tree j on plot i for time period k (either DBH^2 or total ht)

$f(X_{ijk})$ = predicted growth for tree j on plot i for time period k

X_{ijk} = vector of predictor variables

E_{ijk} = random element.

Therefore, the larger the predicted growth the larger the absolute effect of the random element E.

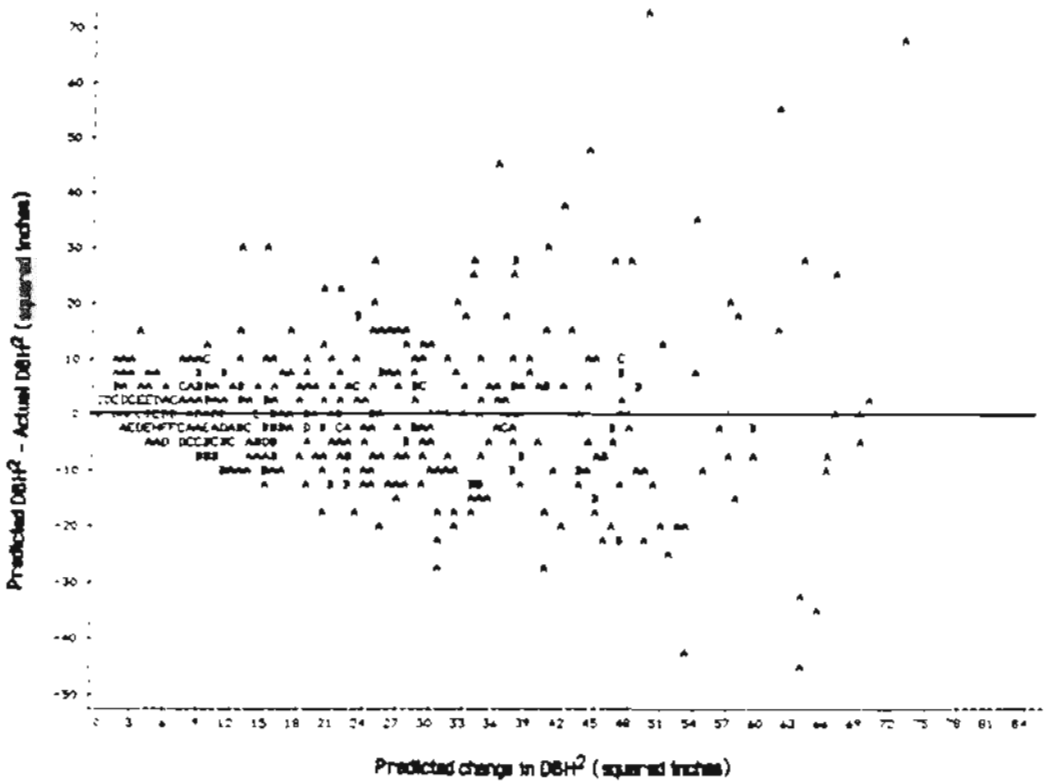

Figure 1. Residuals from DBH^2 growth model plotted on predicted DBH^2 growth for ponderosa pine.

In some cases the predictive model $f(X_{ijk})$ can be shown to systematically over or under estimate actual growth for particular plots, trees, time periods, etc. The uncertainty in predictions may be reduced if a systematic trend can be estimated separately from the growth model,. Consider the following possible components of the random element E:

$$E_{ijk} = \alpha_i + \beta_{ij} + \gamma_k + \delta_{ijk} + \epsilon_{ijk} \qquad [2]$$

where α_i = cluster and plot effect for plot i

β_{ij} = tree effect for tree j on plot i

γ_k = period effect for period k

δ_{ijk} = measurement effect for tree j on plot i for period k

ϵ_{ijk} = unexplained error for tree j on plot i for period k

The tree effects (β) may be due to variations in the response of tree growth due to genetic and historical factors not used by the growth models. The plot and cluster effects (α) may be due to estimation errors in predicting site index and crown competition as well as any inadequacies of site index to reflect actual productivity. Period effects (γ) may be due to variation in the growth environment from year to year (amount and distribution of rainfall, temperature) and other environmental factors not included in the prediction process. The measurement effects (δ) may be due to errors in field measurements or in subsequent data processing. The unexplained error (ϵ) is the error left over after all of the other effects have been taken into account. In cases where plot, cluster, tree, period, or measurement effects can be estimated, more accurate estimates of the mean growth can be obtatined, thus reducing the amount of unexplained variation.

Assuming independence between the components, the sums of squares of E (equation 2) can be broken down as follows:

$$SS_E = SS_{plot} + SS_{tree} + SS_{period} + SS_{meas.} + SS_{unexplained} \qquad [3]$$

where the sums of squares are defined as follows:

SS_{plot} plot (and cluster) effects

SS_{tree} tree effects

SS_{period} period effects

$SS_{meas.}$ measurement errors

$SS_{unexplained}$ unexplained sums of squares

A "clean" breakdown of the element E into it's component parts would require a "crossed" design for the sample data. That is, multiple measurements on multiple trees per plot on multiple plots at multiple points in time. With such data, conventional analysis of variance techniques can be

used to partition the sums of squares shown in equation [3] and to estimate the effects shown in equation [2]. Those effects that cannot be estimated are lumped with the unexplained error

Separate studies can be used to estimate measurement error and thereby reduce the dimension of the problem. However, measurement errors inherent in the data under study were not investigated on a scientific basis. Fortunately, separating measurement and unexplained errors is not very critical since the measurement error should be controlled and remain small. Also, measurement error should be included in the stochastic process rather that be left up to user calibration.

In the stem analysis study, only a subset of the trees on a given plot were felled for stem analysis, with the other trees on the plots being measured for current size and radial increment using an increment borer. Unfortunately, since the stem analysis data set only has time series data for a subset of the trees on the plots, the last four elements of equation [3] cannot be estimated. However, the plot effects were estimated separately and they may be included in CACTOS by calibrating the models directly (**Wensel & Daugherty, 1985**).

Estimates of the plot effects in equation [2] can be separated into cluster and plot-within-cluster effects. For each of the two growth models and six species, a nested ANOVA model was used to separate the variation as follows:

$$\varepsilon_{((h)i)j} = \mu + \alpha_h + \alpha_{(h)i} + e_{((h)i)j} \qquad [4]$$

where

$\varepsilon_{((h)i)j}$ = residual of jth tree on i^{th} plot in h^{th} cluster

μ = overall bias (hopefully zero)

α_h = effect for cluster h

$\alpha_{(h)i}$ = effect for plot i in cluster h

$e_{((h)i)j}$ = unexplained error (due to tree, period and measurement effects)

The $e_{((h)i)j}$'s were then processed to find the standard error and measure of skewness for each species and growth model(**Table 1**). This was done by using the procedure GLM (general linear

model) of the statistical program SAS (**SAS, 1982**). In summary, the cluster and plot effects were significant in most cases with the variance reductions ranging from 5 to 45 percent. Also, the unexplained errors were unbiased with standard errors between 0.20 and 0.36 for height growth and between 0.34 and 0.52 for diameter squared growth. Except for the height growth of the pines, positive values of the corrected third moments were found. Positive skewness of the unexplained errors is expected because the unexplained errors are constrained at the lower end by -1 (corresponding to an actual growth rate of zero) and uncontrained on the upper end. Therefore, the error distributions do not follow a central normal distribution.

Table 1. Standard error ($\hat{\sigma}$) and skewness ($\hat{\theta}$) estimates for DBH^2 and height growth models.

Coefficients	PP	SP	IC	DF	WF	RF
DBH^2 growth						
$\hat{\sigma}$	.46	.34	.49	.44	.49	.52
$\hat{\theta}$	.10	.03	.18	.13	.15	.23
height growth						
$\hat{\sigma}$	.22	.20	.29	.27	.27	.36
$\hat{\theta}$	-.06	-.07	.08	.05	.10	.07

The skewnormal distribution provides good approximations to many near-normal distributions (**Hodges, 1978**) and was chosen to represent the error distributions. The probability density function of the standardized skewnormal is approximated by (to the order of θ^2):

$$\phi_s(z) \doteq \phi(z) - \theta\,\phi'''(z)$$

or

$$\phi_s(z) \doteq \phi(z)\,(1 + \theta\,(z^3 - 3z)) \qquad [5]$$

where

$\phi_s(z)$ = probability density function of the skewed normal distribution,

$\phi(z)$ = probability density function of the standardized normal distribution,

$\phi'''(z)$ = third derivative of $\phi(z)$, and

θ = standardized third moment / 6

Similarly, the cumulative distribution function (cdf) is approximated by:

$$\Phi_s(z) \doteq \Phi(z) - \theta\,\phi''(z)$$

$$\Phi_s(z) \doteq \Phi(z) - \theta\,(z^2 - 1)\,\phi(z) \qquad [6]$$

where

$\Phi_s(z)$ = cumulative distribution of the skewnormal distribution, and

$\Phi(z)$ = cumulative distribution function of the standardized normal distribution.

IMPLEMENTATION

The stochastic technique used in CACTOS must allow the user to replicate a simulation run. That is, the difference between simulation runs must be due to the factors changed and not due to the random process used. This enables the forest manager to estimate the effects of various management regimes without any confounding stochastic effects.

FOREST (**Ek & Monserud, 1974**) uses a random number generator to generate values of E (equation [1]) from a normal distribution, with a mean of 0 and variance equal to the variance of the residuals divided by the average growth rate found in their analysis. This was done for both the DBH^2 and height growth models. While this is a direct method of applying stochastic effects to growth predictions, parallel simulations with different management actions cannot be replicated. Also, the use of a the central normal distribution doesn't reflect the skewed nature of the residuals found in the current study.

PROGNOSIS (**Stage, 1973**) uses a similar method to incorporate stochastic effects for the DBH^2 1/ growth when there are many tree records since "... the effects of any one random deviation on the growth rate of one tree would be blended with many other tree records. Consequently, the stand totals should be quite stable estimates." PROGNOSIS requires 1350 tree records for this method to be implemented. If fewer tree records are entered into the simulator, a record-tripling scheme is used, with each of the new tree records having the same characteristics of the original tree except the tree weights are reduced to 15% and 25% of the original tree. The original tree's weight is reduced to the remaining 60%. The first new tree record represents the slower growing trees (up to the 15th percentile) while the second new tree record represents faster growing trees

1/ PROGNOSIS does not use stochastic effects for its height growth models.

(75th percentile and up). The original tree record represents the medium growing trees (15th to 75th percentiles). Stage assumes that the error distribution is normally distributed and calculates the growth modifiers ($E = z_i$, i=slow,med.,fast) to be the expected value of the regions of the error distribution represented respectively by each of the three similar tree records(See **Figure 2**). PROGNOSIS triplicates each tree record every growth cycle until a limit of 1350 trees is reached whereby the scheme similar to FOREST's is implemented.

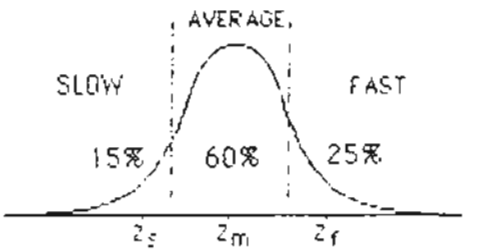

Figure 2. The error distribution breakdown for PROGNOSIS DBH^2 growth model.

CRYPTOS (**Krumland & Wensel, 1980**) uses a record tripling scheme similar to PROGNOSIS,that is applied to both the DBH^2 and height growth models. Independence is assumed between the two growth components and a joint normal error distribution is used in determining the appropriate weights. Further, for computational efficiency, CRYPTOS uses 4 of the 9 possible regions of the joint error distribution (See **Table 2**).

Table 2. Break down of tree weights for CRYPTOS

HEIGHT	DBH^2 GROWTH			
GROWTH	Slow	Med.	Fast	Sum
Slow		.25		.25
Med.	.25		.25	.50
Fast		.25		.25
Sum	.25	.50	.25	1.00

Hence, CRYPTOS quadruples each tree record with each replication having 25% of the original tree record's weight. The values E are computed as the median of each region of the error distributions represented by that new tree record. This method has been coined "pseudo-stochastic" since this scheme incorporates the unexplained variation while not using random errors.

The stochastic scheme employed by CRYPTOS has the desirable property of replicability and is employed jointly between the DBH^2 and HT growth models. However, the assumption of normal errors is not valid in this case.

CACTOS employs a stochastic method similar to CRYPTOS but the errors are assumed to be from a skewnormal distribution with parameters given by **Table 1**. The error distribution for each growth model and species was divided into three regions -- slow, medium and fast growth, as illustrated in **Figure 3**. Using the

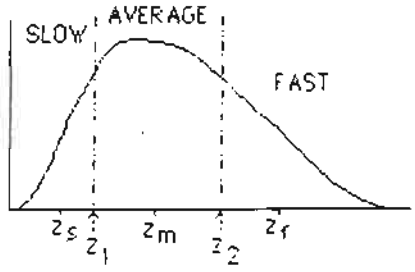

Figure 3. Error regions for CACTOS growth models.

distribution function for the skewnormal, Φ_s, the area of these regions are computed by the following relationships:

$$A(s) = \Phi_s(z_1)$$

$$A(m) = \Phi_s(z_2) - \Phi_s(z_1)$$

$$A(f) = 1 - \Phi_s(z_2)$$

Further, independence was assumed between the height and diameter growth error terms because no data was available to investigate alternative joint density distributions.

The tree record weights and growth modifiers are computed as follows. First z_1, the dividing point between the slow and medium growing trees, was found such that $A(s) = {}^1/_6$, or

$$\Phi_s(z_1) = 0.1667$$

Hence, the marginal tree record weight for the tree records representing this region is 1/6 of the original tree record weight with the growth modifier z_s being equal to the expected value of the distribution within this region. Second, it was desirable to have one of the *tupled* Otree records to grow as average prediction (growth modifier of 0). Thus, z_2 was found such that the expected value of the distribution between z_1 and z_2 was 0. Third, the expected value of the distribution between z_2 and $+\infty$ for the growth modifier for the fast growing trees, z_f, was computed. Similarly, the marginal tree record weight for this region is the area of the distribution within this region times

the original tree record weight. A FORTRAN computer program to compute the bounds of the region, z_1 and z_2, and the means of the slow and fast regions, z_s and z_f, is available from the authors. The program uses equation [5] for the density of the skewnormal and the areas are consistent with the approximations given by equation [6].

The computed marginal tree record weights were then used to find the tree record weights for each of the new quintupled tree records. The means of the slow, medium, and fast regions are given in **Table 3** and the relative tree weights are given in **Table 4**. Note that for each growth model the sum of the marginal tree record weights add up to the original tree record's weight and the sum of the growth modifiers times the marginal tree record weights for each region adds up to 0.

Table 3. Dividing points and means of slow, medium and fast regions for growth model.

Coefficients	PP	SP	IC	DF	WF	RF
DBH^2 growth						
z_1	-.973	-.969	-.976	-.974	-.975	-.978
z_2	1.211	1.029	1.512	1.311	1.386	1.740
z_s	-1.354	-1.456	-1.238	-1.311	-1.281	-1.167
z_m	0.	0.	0.	0.	0.	0.
z_f	1.852	1.594	2.179	1.973	2.057	2.365
height growth						
z_1	-.963	-.962	-.972	-.970	-.973	.-971
z_2	0.863	.847	1.152	1.075	1.211	1.125
z_s	-1.586	-1.601	-1.383	-1.427	-1.354	-1.398
z_m	0.	0.	0.	0.	0.	0.
z_f	1.333	1.308	1.775	1.663	1.852	1.735

Table 4. Break down of tree weights by species for CACTOS record quintupling.

Ponderosa Pine

HEIGHT	DBH² GROWTH			
GROWTH	Slow	Med	Fast	Sum
Slow		.1667		.1667
Med.	.1667	.3465	.1218	.6350
Fast		.1983		.1983
Sum	.1667	.7115	.1218	1.0000

Sugar Pine

HEIGHT	DBH² GROWTH			
GROWTH	Slow	Med	Fast	Sum
Slow		.1667		.1667
Med.	.1667	.3104	.1522	.6293
Fast		.2040		.2040
Sum	.1667	.6811	.1522	1.0000

Incense Cedar

HEIGHT	DBH² GROWTH			
GROWTH	Slow	Med	Fast	Sum
Slow		.1667		.1667
Med	.1667	.4421	.0947	.7035
Fast		.1298		.1298
Sum	.1667	.7386	.0947	1.0000

Douglas Fir

HEIGHT	DBH² GROWTH			
GROWTH	Slow	Med	Fast	Sum
Slow		.1667		.1667
Med.	.1667	.4129	.1107	.6903
Fast		.1430		.1430
Sum	.1667	.7226	.1107	1.0000

White Fir

HEIGHT	DBH² GROWTH			
GROWTH	Slow	Med	Fast	Sum
Slow		.1667		.1667
Med.	.1667	.4410	.1038	.7115
Fast		.1218		.1218
Sum	.1667	.7295	.1038	1.0000

Red Fir

HEIGHT	DBH² GROWTH			
GROWTH	Slow	Med	Fast	Sum
Slow		.1667		.1667
Med.	.1667	.4501	.0822	.6990
Fast		.1343		.1343
Sum	.1667	.7511	.0822	1.0000

APPLICATION

A mixed-conifer stand with a per acre stocking of 295 trees, 265 square feet of basal area, and 6750 cubic feet of volume is used to illustrate the process of generating pseudo-stochactic quintuples. Table 5 gives the size and growth information on the first 5 of the trees in the tree list for the stand. This shows, for example, that the 5-year predicted DBH growth is 0.84 inches while the predicted height growth is 5.6 feet.

Table 5. Size and Predicted growth for first 5 trees (of 33) on a one-fifth acre plot (TPA = trees per acre).

no.	species	DBH	total ht	crown ratio	TPA	5-year growth ΔDBH	Δht
1	WF	14.4	85	.39	5.00	.84	5.6
2	PP	24.5	49	.55	5.00	.65	4.8
3	PP	18.2	84	.50	5.00	.75	5.6
4	IC	14.8	53	.41	5.00	.53	2.6
5	PP	22.1	87	.32	5.00	.65	5.7

The DBH and height growth estimates for the quintupled record are then computed as follows From Table 1 for white fir (WF) the standard deviations are

$$\hat{\sigma}_D = 0.49 \quad \text{and} \quad \hat{\sigma}_H = 0.27$$

and from Table 3 for white fir the skewnormal deviations are

	DBH	Height
z_{slow}	-1.281	-1.354
z_{med}	0.	0.
z_{fast}	2.057	1.852

The values of (1+E) for equation 1 are then computed as

$$1 + E = 1 + z\hat{\sigma}$$

yielding the "slow" diameter growth estimate of

$$\Delta D_{slow} = \Delta D\,(1 + E_{slow})$$
$$= 0.84\ [1 + (-1.281)(0.49)]$$
$$= 0.84\ [\ 0.372\ ]$$
$$= 0.32 \text{ inches}$$

Following this process, the remaining values of (1+E) for the first tree are given as follows

	DBH	Height
slow	0.372	0.634
med.	0.	0.
fast	2.008	1.500

Multiplying these values of (1+E) by the DBH and height growth rates predicted by CACTOS, 0.84 and 5.6 feet, respectively, yields the quintuple growth rates given in **Table 6** for the first 15 trees. The order of the records within each quintuple is as follows:

(1) ΔD_{med} , ΔH_{med}

(2) ΔD_{slow}, ΔH_{med}

(3) ΔD_{fast}, ΔH_{med}

(4) ΔD_{med}, ΔH_{slow}

(5) ΔD_{med} , ΔH_{fast}

Table 6. Predicted DBH and height after a single 5-year growth cycle using quintipuled growth records - first 5 trees (of 33) on a one-fifth acre sample plot.

no.	sp	DBH	total ht	crown ratio	TPA	(1 + E) DBH	(1 + E) ht
1	WF	15.2	91.	0.39	2.21	1.000	1.000
	WF	14.7	91.	0.39	0.83	0.372	1.000
	WF	16.0	91.	0.39	0.52	2.008	1.000
	WF	15.2	89.	0.39	0.83	1.000	0.634
	WF	15.2	93.	0.40	0.61	1.000	1.500
2	PP	25.1	54.	0.54	1.73	1.000	1.000
	PP	24.7	54.	0.53	0.83	0.377	1.000
	PP	25.7	54.	0.55	0.61	1.852	1.000
	PP	25.1	52.	0.55	0.83	1.000	0.651
	PP	25.1	55.	0.54	0.99	1.000	1.293
3	PP	19.0	90.	0.49	1.73	1.000	1.000
	PP	18.5	90.	0.48	0.83	0.377	1.000
	PP	19.6	90.	0.50	0.61	1.852	1.000
	PP	19.0	88.	0.50	0.83	1.000	0.651
	PP	19.0	91.	0.49	0.99	1.000	1.293
4	IC	15.3	56.	0.40	2.21	1.000	1.000
	IC	15.0	56.	0.40	0.83	0.393	1.000
	IC	15.9	56.	0.40	0.47	2.068	1.000
	IC	15.3	55.	0.40	0.83	1.000	0.599
	IC	15.3	57.	0.39	0.65	1.000	1.515
5	PP	22.7	93.	0.32	1.73	1.000	1.000
	PP	22.3	93.	0.32	0.83	0.377	1.000
	PP	23.3	93.	0.33	0.61	1.852	1.000
	PP	22.7	91.	0.32	0.83	1.000	0.651
	PP	22.7	94.	0.32	0.99	1.000	1.293

The quintuple estimates for the other trees follows in a similar manner using the appropriate values of the standard error ($\hat{\sigma}$) and skewnormal deviation (z) from **Tables 1 and 3**, respectively.

The tree expansion factors for each of the trees shown in **Table 5** is 5 trees per acre before quintupling. The expansions for the tree records after quintupling (**Table 6**) are obtained by multiplying the original expansion by the weights given for each species in **Table 4**. Remember that the sum of the new tree weights must be equal to the original tree weight.

The impact of implementing pseudo-stastic variation in the CACTOS model can be illustrated by a 30-year simulation for the sample plot referred to above. The basal area growth for 30-year period is given in **Table 7**, with no thinning, with thinning from below, and with thinning from above. Estimates from simulations with and without pseudo-stochastic effects are shown.

Table 7. Basal area growth for 30 years with and without pseudo-stochastic effects for varying thinning alternatives. (All thinning at the end of the first 5-year cycle.)

	pseudo-stochastic effects		
	no	yes	% diff
no thinning	128.8	130.9	1.6
thin 70 ft^2 from below	105.3	107.7	2.2
thin 70 ft^2 from above	122.3	124.1	1.5
thin 140 ft^2 from below	82.3	83.7	1.7
thin 140 ft^2 from above	117.7	119.7	1.7

In each case using pseudo-schostatic effects increased the predicted growth rates. Differences also exist in the predicted volume growth rates.

DISCUSSION

The tupling of tree records to represent the unexplained variation in tree growth predictions is a useful tool for tree growth prediction. Quintupling, as used here has the intuitive advantage that the growth rates for one of the tupled records are unchanged. Also, in contrast to previous tupling schemes, the current scheme recognizes the skewed nature of the unexplained variation in the growth predictions. Estimators from the skewnormal distribution are used to represent this unexplained variation in forming the quintuples.

Short term predictions without management intervention can be done without the use of pseudo-stochastic variation. For such a small period, there will be little or no difference in the predictions. Thus the CACTOS model can be used to update timber inventories without loss of accuracy and without affecting the integrity of the original records. However, for long term predictions and/or simulations with management interventions, the use of pseudo-stochastics is advised.

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