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TREE TAPER MODELS FOR MAJOR COMMERCIAL CALIFORNIA CONIFERS

by

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ABSTRACT

Equations are developed to estimate the stem taper of inside-bark diameters from breast height to the tip. Separate equations are developed to estimate the diameter inside bark at the stump. Equations are fitted and tested on separate halves of a data set composed of tree taper data from previous studies, and industry and U.S.D.A. Forest Service surveys. This data set extends from southern California to the National Forests of southern Oregon.

After an extensive examination of existing taper equations, two taper equations were selected for further fitting to the present data set, one by Biging and one by Wensel and Krumland. Coefficients for both equations are given for the 8 conifer species examined. Because of a lower residual sum of squares, and a lack of correlation of the residual with the available predictors, the Wensel and Krumland equation is recommended for use.

To distinguish this equation and its new coefficients from those developed by Wensel and Krumland, the Wensel and Krumland model with the new coefficients fitted here is referred to as the Wensel and Olson model, both for upper stem taper and, separately, for inside-bark stump diameter estimates.

An examination of the possible need for "local" modifications in the taper coefficients was made. No correlation was found between the residuals and combinations of latitude, elevation, and site index. As a result, the Wensel and Olson coefficients are recommended without local adjustments.

The volume of standing trees must be estimated from measurements on these trees for general forest land planning (harvest scheduling) and for appraisal. Particularly for appraisal purposes volumes must be estimated by log quality (grade) classes with appropriate deductions for defect. Whether one estimates volume in cubic or board foot scale, this flexible volume estimation process is made possible by use of tree taper equations - equations that predict the diameter inside-bark of a tree at any height. Given these estimated diameters, either board foot or cubic foot

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volumes can be computed for any section. This process of tree volume determination is beginning to replace the scaling of felled trees for U.S. Forest Service and private timber sales requiring a new look at the procedures used for such estimates.

The U. S. Forest Service - Region 5 (California) has been using tree taper equations computed by Stadelman (1986) for this purpose while industry members of the California Forest Research Association have been using equations by Biging (1984) that have been incorporated in CACTOS, the California Conifer Timber Output Simulator (Wensel, Daugherty, and Meerschaert, 1986). These equations have different forms and they produce different predictions. Further, there is a "wealth" of tree taper data available from various sources, but existing taper models were fitted over a small subset of the data presented in this paper.

The objective here is to test these and other local equations for estimating tree taper and, if necessary, develop recommendations for both the best model form to use and the coefficients to use in each region within the state. The various equations considered, the data sets used for testing, and the test results of all equations against all data sets will be presented. A recommended model form is presented along with a discussion of attempts to localize the predictions.

Species considered are as follows:

Douglas-fir	*Pseudotsuga menziesii* [Mirb.] Franco
ponderosa pine	*Pinus ponderosa* Dougl. ex Laws.
Jeffrey pine	*P. jeffreyi* Grev. & Balf.
lodgepole pine	*P. contorta* Dougl. ex Loud.
sugar pine	*P. lambertiana* Dougl.
white fir	*Abies concolor* [Gord. & Glend.] Lindl. ex Hildebr.
red fir	*A. magnifica* A. Murr.
incense-cedar	*Calocedrus decurrens* [Torr.] Florin

I. Taper equations considered

This analysis started with the examination of the performance of existing tree taper models with their published coefficients for west coast conifers. The equations considered are shown in Table 1. These equations will be discussed in the order that they were developed.

Predictions Above Breast Height

The Stadelman taper equation, *STA*, was developed by Chuck Stadelman for the U.S. Forest Service Region 5 (California) by fitting a polynomial to each of 4 tree diameter classes: DBH < 18 inches, 18 ≤ DBH < 30 inches, 30 ≤ DBH < 40 inches, and DBH ≥ 40 inches.

Further, the Stadelman equations used information on the relative position in the tree in the prediction. With coefficients fitted in 1977, these equations have formed the basis for volume estimation in the National Forest inventories and appraisals in California. In addition to any problems the equations might have producing accurate predictions in any one diameter class, the Stadelman equations are not designed to produce a smooth transition in the predictions as a tree grows from one size class to the next.

The Wensel and Krumland (1983) equation, *W&K*, was developed for redwood and Douglas-fir on California's north coast. The coefficients were fitted to measurements of both standing trees and felled trees. The coefficients for redwood and Douglas-fir form the basis for tree taper computations in CRYPTOS, the Cooperative Redwood Yield Project's Timber Output Simulator (Wensel, Krumland, and Meerschaert, 1987).

For his equation, Biging (1984) developed a restricted version of the *W&K* model by fixing one of the coefficients so that it no longer changes with tree size, the *BIG* model. This reduces the number of parameters to be estimated and makes it possible to integrate the taper equation for volume, making the taper and volume equations compatible. Biging fitted his coefficients to measurements taken from trees felled for stem analysis as part of the work of the Northern California Forest Yield Cooperative.

Biging also fitted coefficients to a segmented polynomial equation proposed by Max and Burkhart (1976), the *M&B* model. Biging found both the *BIG* and *M&B* equations to perform about the same, but Biging's model is more parsimonious. It uses only two fitted coefficients for each species while the *M&B* model requires as many as 6 coefficients, depending on how many are fixed in any given solution. Biging's coefficients for both his and the *M&B* models were considered here.

Amidon (1984) published an equation that was examined here. However, apparent errors in the equation formulation and/or coefficients could not be resolved so that meaningful estimates could not be computed[2]. Thus, formal comparisons of the accuracy of Amidon's predictions are not presented here.

Working in southern Oregon, Walters and Hann (1986) developed a two-segmented polynomial model that adds crown length as a predictor, the *W&H* model. It has the potential of being more sensitive to variations in stem taper due to stand density.

McTague and Stansfield (1988) working in Arizona developed a set of volume and tree taper equations for southwestern ponderosa pine. Their functions are straight-forward power functions on tree DBH, and Girard form class. As such, they are particularly easy to fit and to apply. Their taper equation, the *McT* model, is given in Table 1.

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[2] These errors were also detected by Greg Biging, Robert Curtis, and David Bruce (personal comment).

Finally, two equations from British Columbia for Douglas-fir were considered (Kozak, Munro, and Smith, 1969; Kozak, 1988). The first is a rather simple two-parameter function on relative height, and the second is a more involved exponential model including the relative height to the taper inflection point, the *KOZ* model in Table 1.

Table 1. Taper equations evaluated, where dib is the predicted diameter inside-bark.

Symbols

dib	diameter inside bark
dbhib	diameter breast height inside bark
DBH	diameter at breast height outside bark (4.5')
HT	height to the point of interest
THT	total height (to the tip except in the Wensel & Krumland model)

Amidon (1984)

$$\text{dib} = b_1\,\text{DBH}\left(\frac{\text{THT} - \text{HT}}{\text{THT} - \text{bh}}\right) + b_2\frac{(\text{THT}^2 - \text{HT}^2)(\text{HT} - \text{bh})}{\text{THT}^2} \tag{1}$$

where bh is breast height (1.37m).

Biging (1984)

$$\text{dib} = \text{DBH}\left\{b_1 + b_2 \ln\left[1 - \lambda\left(\frac{\text{HT}}{\text{THT}}\right)^m\right]\right\} \tag{2}$$

where $\lambda = 1 - \exp\left(\frac{-b_1}{b_2}\right)$

and $m = \frac{1}{3}$.

Kozak, et.al. (1969)

$$\frac{\text{dib}^2}{\text{DBH}^2} = b_1\frac{(\text{HT}-\text{THT})}{\text{THT}} + b_2\frac{(\text{HT}^2 - \text{THT}^2)}{\text{THT}^2} \tag{3}$$

Kozak (1988)

$$\text{dib} = \left\{a_0\,\text{DBH}^{a_1}\,a_2\text{DBH}\right\}\left\{X^{b_1 Z^2 + b_2 \ln(Z + 0.001) + b_3\sqrt{Z} + b_4 e^Z + b_5\left(\frac{\text{DBH}}{\text{THT}}\right)}\right\} \tag{4}$$

where: $X = \left(\frac{1 - \sqrt{\frac{\text{HT}}{\text{THT}}}}{1 - \sqrt{p}}\right)$; $p = \left(\frac{\text{HI}}{\text{THT}}\right) * 100$;

HI = height of the inflection point from ground; and $Z = \frac{\text{HT}}{\text{THT}}$

Max & Burkhart (1976)

$$\frac{\text{dib}^2}{\text{DBH}^2} = b_1\left(\frac{\text{HT}}{\text{THT}} - 1\right) + b_2\left(\frac{\text{HT}^2}{\text{THT}^2} - 1\right) + b_3\left(a_1 - \frac{\text{HT}}{\text{THT}}\right)^2 I_1 + b_4\left(a_2 - \frac{\text{HT}}{\text{THT}}\right)^2 I_2 \tag{5}$$

where $I_i = 1$ for $\frac{\text{HT}}{\text{THT}} \le a_i$, $I_i = 0$ for $\frac{\text{HT}}{\text{THT}} > a_i$ and $i = 1,2$

Table 1. (cont).

McTague and Stansfield (1988)

$$dib = \frac{.0692939\ (THT - HT)^{0.732737}\ DBH^{0.977953}\ F^{0.617250}}{THT^{0.729358}} \tag{6}$$

where: $F = 77.73 + .000056598\ DBH^2\ THT - \frac{40042.8}{DBH^2\ THT}$

Stadelman (1986)

$$dib = a + b\ DBH^2 + c\ DBH\ L + d\ DBH^2\ L + e\ L + f\ Z + g\ X \tag{7}$$

where $L = \frac{THT-HT}{THT}$, $X = THT - H$, $Z = \ln$ (log position) and a, b, c, d, e, f, g are coefficients.

Walters & Hann (1986)

$$Y = Z_0 + \left[b_{10} + b_{11}\left(\frac{THT - 4.5}{DBH}\right) + b_{12}\left(\frac{THT - 4.5}{DBH}\right)^2\right] Z_1 + b_{20}\ Z_2 \tag{8}$$

where Y = dib/dbhib

$$Z_0 = 1.0 - X + I_2\left\{X + I_1\left[\left(\frac{X - 1.0}{k_1 - 1.0}\right)\left(1.0 + \frac{k_1 - X}{k_1 - 1}\right) - 1.0\right]\right\} - (X - 1)(X - I_2\ X)$$

$$Z_1 = I_2\left[X + I_1\left(\frac{X - 1.0}{k_1 - 1.0}\right)\left(X + k_1\frac{k_1 - X}{k_1 - 1}\right) - X\right] - (X - 1)(X - I_2\ X);$$

$$Z_2 = I_2\left\{X^2 + I_1\left[k_1\left(\frac{X - 1.0}{k_1 - 1.0}\right)\left(2\ X - k_1 + k_1\left(\frac{k_1 - X}{k_1 - 1.0}\right)\right) - X^2\right]\right\};$$

$k_1 = \left(\frac{\alpha_1\ CB - 4.5}{THT - 4.5}\right)$; $X = \frac{HT - 4.5}{THT - 4.5}$; CB = height to live crown base;

$$I_1 = \begin{Bmatrix} 0.0 \text{ when } 0 \le X \le k_1 \\ 1.0 \text{ when } k_1 < X \le 1 \end{Bmatrix}; \text{ and}$$

$$I_2 = \begin{Bmatrix} 0.0 \text{ when } k_1 \le 0.0 \\ 1.0 \text{ when } k_1 \ge 0.0 \end{Bmatrix}.$$

Wensel and Krumland (1983)

$$dib = DBH\left\{c_0 - f\ln\left\{1 - R^{c_1}\left[1 - \exp\left(\frac{c_0}{f} - \frac{d_m}{DBH\ f}\right)\right]\right\}\right\} \tag{9}$$

where $R = \frac{HT - 1}{THT - 1}$, d_m = merchantable top diameter inside bark corresponding to point THT, and $f = c_2 + c_3\ DBH + c_4\ THT$

Predictions Below Breast Height

Predictions of tree diameters at points below breast height are useful for estimating total biomass. However, they are not needed for estimating board foot volume since the scaling volume of the first log is at the small end of the log, usually at 16.5, 20.5, or 33.0 feet above the stump. Researchers have used separate models for predicting the inside-bark diameter below breast height. Walters and Hann (1986) present methods for predicting diameter below breast height either when the height to crown base is known or when it must be predicted. In each case, the diameter is computed as a function of a predicted diameter inside bark at breast height (4.5 feet) and at a height of 1.0 feet (Walters, Hann, and Clyde 1985). Since we presume here that only one tree diameter will be available, usually at breast height, this model was not tested here.

Wensel and Krumland used the stump diameter prediction model:

$$d_s = (1 - b_0 X)\, DIA_i\, e^{\,b_1(h_i - h_s)}$$

where d_s is the diameter at the stump at height h_s, DIA_i is the measured diameter inside or outside bark at height h_i, X is an indicator variable equal to 0 or 1, if the DIA_i is inside- or outside-bark, respectively, and b_0 and b_1 are coefficients. In most applications DIA_i is equal to DBH and h_i = 4.5 feet.

II. Data Sources

Taper data for this study came from previous studies at the University of California, Berkeley, and from other researchers in California and Oregon. These data are of 2 types, either from stem analysis of felled trees or from optical measurements on standing trees with an optical dendrometer. In pooling the data for taper analysis, the origin of the data elements was preserved so we could look for differences in the performance of the various models to the individual data sets. Tables 2, 3a, and 3b present the numbers of trees for each species by data set for data from previous research projects and forest industry (Table 2) and from National Forest surveys (Tables 3a and 3b). These data sets are described briefly below.

BULLETIN 1883: These data were collected in 1976 from standing trees using optical dendrometers (see Table 2). The data set is made up of data collected from lands then managed by American Forest Products, Southern Pacific Land Company, the Plumas National Forest, and the Stanislaus National Forest. The data were used to develop volume tables for young-growth conifers (Wensel 1977).

COOP: The Coop data come from a stem analysis study by the members of the Northern California Forest Yield Cooperative (see Table 2) on industry forest lands in the Sierra, Southern Cascade, Shasta-Trinity and Mendocino regions of California. This Coop research group includes UC Berkeley, private industry, and public forestry agencies; it was formed to produce growth prediction models for California conifers. The inside-bark measurements in these data were used for the initial growth models for CACTOS (Wensel, Meerschaert, and Biging, 1986; Wensel and Robards, 1989) and to produce stem taper models (Biging 1984).

DOLPH: These data resulted from two stem analysis studies by Leroy Dolph, USDA Forest Service (see Table 2). The first was a study of conifer trees in California's Sierra Nevada and was the source for the USFS tree taper and growth work. These data are described by Dolph (1988). The second study centered on red fir in northern California and southern Oregon and the data are described by Dolph (1989).

LEVITAN: Observations on one hundred incense-cedar trees from the Klamath, Lassen, and Mendicino National Forests were collected by Jack Levitan in 1990. Using an optical dendrometer, he obtained stem taper data for incense-cedar in areas where it is a common commercial species yet, was missing from previously collected data (see Table 2).

LINDQUIST: These data for Douglas-fir were received from James Lindquist. The data were collected in Northern Californian National Forests using an optical dendrometer (see Table 2).

MICH-CAL: These data were collected by scaling felled trees and were used by the Michigan-California Lumber Company to compute new coefficients for the taper and volume equations developed by Biging (1984) (see Table 2).

MILL STUDY: These stem taper data came from a series of utilization studies conducted in California by the USFS PNW Research Station (see Table 2). Trees were felled and scaled in the field, then scaled again in the mill before being sent through the mill for conversion. The primary purpose of the collection was for lumber recovery to make adjustments in the timber sale appraisal process and get the highest value.

OLIVER: These data were received from Bill Oliver, USDA Forest Service (see Table 2). They were collected with an optical dendrometer from plots located on National Forest lands of Northern California to determine the growth responses to silviculture treatments (Oliver 1988).

TARIF: The TARIF data were collected as a cooperative effort prior to the forming of the Northern California Forest Yield Cooperative (see Table 2). The northern California companies and agencies involved included Beaty and Associates, Crane Mills, Diamond International, Fibreboard, Fruit Growers Supply, The Hearst Corporation, International Paper Company, Kimberly-Clark, Louisiana Pacific, Paul Bunyan, Publishers Forest Products, Soper-Wheeler Company, Southern Pacific Land Company, USDA Forest Service, and US Plywood. The data were collected from standing trees using optical dendrometers for the purpose of developing an "access table" for a northern California tarif system. Biging (1981) examined these data for a previous version of his taper equation.

USDA Forest Service - Region 5: These data were collected from National Forests in California, with most of the data coming from the northern forests (see Table 3a). Measurements were made using optical dendrometers for various regional studies and aggregated by Charles Stadelman, USDA Forest Service, San Francisco[3].

USDA Forest Service - Region 6: The stem taper data come felled-tree measurements from trees randomly chosen from clearcut operations in the National Forests in southern Oregon (see Table 3b). The purpose of the data collection was to adjust regional gross volume estimates for inventory to reflect local utilization. There were no formal or internal publications from these data. The data were received with the cooperation of Ralph Johnson, John Teply, and Susan Willits, USDA Forest Service.

[3] The data received from Stadelman were edited by the authors to remove duplicate data from related computer files.

Table 2. Number of trees by source, species, and size classes.

Species	Size class	Bull. 1883	COOP	Dolph	Lindquist Levitan†	Mich-Cal	Mill Study	Oliver	Tarif
DF	5 "- 10 "	-	48	16	65	5	-	-	28
	10 "- 20 "	58	117	30	380	147	58	-	99
	20" - 30"	34	38	3	298	217	76	-	68
	30" - 40"	28	2	-	96	95	67	-	32
	40" - 50"	12	-	-	11	42	71	-	13
	50" +	-	-	-	1	23	65	-	1
PP	5 "- 10 "	-	27	35	-	1	28	93	7
	10 "- 20 "	36	135	84	-	244	151	182	184
	20" - 30"	36	86	27	-	437	129	26	140
	30" - 40"	19	7	1	-	219	45	3	40
	40" - 50"	4	-	-	-	49	22	-	9
	50" +	1	-	-	-	7	15	-	2
SP	5 "- 10 "	-	5	16	-	1	-	-	5
	10 "- 20 "	12	41	34	-	151	-	-	47
	20" - 30"	14	36	12	-	329	10	-	53
	30" - 40"	14	2	-	-	226	25	-	37
	40" - 50"	5	-	-	-	80	26	-	14
	50" +	3	-	-	-	17	24	-	6
JP	5 "- 10 "	-	-	6	-	-	-	-	-
	10 "- 20 "	-	-	18	-	-	48	-	-
	20" - 30"	-	-	6	-	-	7	-	-
	30" - 40"	-	-	-	-	-	-	-	-
	40" - 50"	-	-	-	-	-	-	-	-
	50" +	-	-	-	-	-	-	-	-
LP	5 "- 10 "	-	-	2		-	-	-	-
	10 "- 20 "	-	-	5	-	-	-	-	-
	20" - 30"	-	-	-	-	-	-	-	-
	30" - 40"	-	-	-	-	-	-	-	-
	40" - 50"	-	-	-	-	-	-	-	-
	50" +	-	-	-	-	-	-	-	-
WF	5 "- 10 "	-	67	107	-	10	54	15	19
	10 "- 20 "	47	208	296	-	888	296	177	178
	20" - 30"	44	82	45	-	1232	102	90	209
	30" - 40"	19	4	2	-	463	23	3	48
	40" - 50"	1	-	-	-	83	30	-	4
	50" +	-	-	-	-	12	21	-	-
RF	5 "- 10 "	-	2	66	-	-	-	-	1
	10 "- 20 "	54	27	347	-	-	52	-	112
	20" - 30"	63	14	134	-	-	49	-	121
	30" - 40"	38	-	6	-	-	37	-	46
	40" - 50"	7	-	-	-	-	20	-	8
	50" +	-	-	-	-	-	13	-	-
IC	5 "- 10 "	-	25	101	3	-	-	-	5
	10 "- 20 "	13	60	75	31	132	27	-	35
	20" - 30"	23	10	11	30	192	39	-	65
	30" - 40"	10	1	1	25	110	48	-	33
	40" - 50"	3	-	-	7	39	46	-	9
	50" +	-	-	-	2	13	11	-	7

† Douglas-fir data is from Jim Lindquist and incense- cedar data is from Jack Levitan.

Table 3a. Number of trees by source, species, and size classes for USFS Region 5 National Forests.

Species	Size class	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
DF	5 "- 10 "	-	7	17	7	-	5	-	-	5	-	-	-	-	-	-
	10 "- 20 "	18	24	75	46	-	60	-	-	46	12	-	2	-	-	-
	20" - 30"	19	23	73	44	-	24	-	-	53	11	-	2	-	-	-
	30" - 40"	7	3	38	17	-	-	-	-	11	2	-	1	-	-	-
	40" - 50"	-	3	41	-	-	-	-	-	11	3	-	-	-	-	-
	50" +	-	3	37	-	-	-	-	-	9	2	-	1	-	-	-
PP	5 "- 10 "	-	-	28	9	12	53	-	-	11	-	-	-	-	-	6
	10 "- 20 "	18	-	103	28	31	280	-	-	102	64	-	35	11	-	99
	20" - 30"	20	-	96	24	36	123	-	-	94	93	-	34	4	-	120
	30" - 40"	6	-	14	13	21	20	-	-	44	45	-	25	7	-	128
	40" - 50"	-	-	-	-	8	8	-	-	21	18	-	11	5	-	107
	50" +	-	-	-	-	4	1	-	-	3	6	-	3	-	-	68
SP	5 "- 10 "	-	6	10	6	-	5	-	-	7	-	-	-	-	-	8
	10 "- 20 "	-	28	52	36	-	13	-	-	26	17	-	3	10	-	16
	20" - 30"	-	10	45	28	-	4	-	-	13	13	-	4	12	-	11
	30" - 40"	-	-	17	13	-	-	-	-	5	7	-	14	9	-	16
	40" - 50"	-	-	-	-	-	-	-	-	5	7	-	7	4	-	15
	50" +	-	-	-	-	-	-	-	-	3	4	8	4	5	-	10
JP	5 "- 10 "	-	-	-	-	-	-	-	10	-	-	14	-	6	-	-
	10 "- 20 "	-	-	-	-	-	-	-	26	1	2	63	1	49	21	79
	20" - 30"	-	-	1	-	-	-	-	20	5	1	378	5	55	28	94
	30" - 40"	-	-	-	-	-	-	-	13	1	-	853	12	47	23	68
	40" - 50"	-	-	-	-	-	-	-	-	4	-	430	8	19	27	48
	50" +	-	-	-	-	-	-	-	-	1	-	53	2	3	9	24
LP	5 "- 10 "	-	-	-	1	-	8	-	-	-	-	-	-	-	-	-
	10 "- 20 "	-	-	-	25	-	23	-	-	5	31	-	21	18	-	-
	20" - 30"	-	-	-	16	-	18	-	-	2	26	-	10	15	-	-
	30" - 40"	-	-	-	-	-	-	-	-	2	10	-	5	10	-	-
	40" - 50"	-	-	-	-	-	-	-	-	-	3	-	-	-	-	-
	50" +	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
WF	5 "- 10 "	-	5	19	6	14	27	-	10	13	-	-	-	-	-	7
	10 "- 20 "	9	39	54	33	21	154	16	49	111	51	-	27	48	-	15
	20" - 30"	11	31	48	34	9	41	23	48	94	34	-	36	70	-	14
	30" - 40"	2	26	30	15	5	1	8	17	43	22	-	23	66	-	17
	40" - 50"	-	-	-	-	7	-	-	-	16	8	-	17	26	-	14
	50" +	-	-	-	-	1	-	1	-	5	9	-	13	8	-	1
RF	5 "- 10 "	-	-	8	8	-	7	-	-	-	-	-	-	-	-	-
	10 "- 20 "	-	1	28	37	-	23	1	-	5	8	-	3	20	-	-
	20" - 30"	-	-	22	35	-	4	2	-	18	19	-	12	22	-	-
	30" - 40"	-	-	13	21	-	-	1	-	15	12	-	22	15	-	-
	40" - 50"	-	-	-	-	-	-	-	-	3	19	-	18	21	-	-
	50" +	-	-	-	-	-	-	-	-	5	11	-	7	9	-	-
IC	5 "- 10 "	-	9	-	-	-	1	-	-	11	-		-	-	-	6
	10 "- 20 "	-	25	-	-	-	2	-	-	58	31	-	10	17	-	12
	20" - 30"	-	9	-	-	-	-	-	-	55	19	-	9	16	-	15
	30" - 40"	-	9	-	-	-	-	-	-	35	14	-	6	12	-	14
	40" - 50"	-	-	-	-	-	-	-	-	30	6	-	8	13	-	3
	50" +	-	-	-	-	-	-	-	-	19	6	-	1	8	-	6

(1) Mendicino N.F.
(2) Six Rivers N.F.
(3) Klamath N.F.
(4) Shasta-Trinity N.F.
(5) Modoc N.F.
(6) Lassen N.F.
(7) Plumas N.F.
(8) Tahoe N.F.
(9) Eldorado N.F.
(10) Stanislaus N.F.
(11) Inyo N.F.
(12) Sierra N.F.
(13) Sequoia N.F.
(14) Los Padres N.F.
(15) San Bernardino N.F.

Table 3b. Number of trees by National Forest, species, and size classes for USDA Forest Service Region 6.

Species	Size class	Fremont N.F.	Rouge River N.F.	Siskiyou N.F.	Winema N.F.
DF	5 "- 10 "	-	-	-	-
	10 "- 20 "	-	-	24	-
	20" - 30"	-	-	49	-
	30" - 40"	-	-	51	-
	40" - 50"	-	-	39	-
	50" +	-	-	16	-
PP	5 "- 10 "	-	-	-	-
	10 "- 20 "	61	-	-	42
	20" - 30"	85	-	-	115
	30" - 40"	23	-	-	32
	40" - 50"	3	-	-	2
	50" +	-	-	-	1
LP	5 "- 10 "	21	-	-	20
	10 "- 20 "	166	-	-	140
	20" - 30"	6	-	-	3
	30" - 40"	-	-	-	-
	40" - 50"	-	-	-	-
	50" +	-	-	-	-
WF	5 "- 10 "	-	-	-	-
	10 "- 20 "	41	-	-	96
	20" - 30"	100	-	-	63
	30" - 40"	27	-	-	24
	40" - 50"	1	-	-	-
	50" +	-	-	-	-
RF	5 "- 10 "	-	-	-	-
	10 "- 20 "	-	34	-	65
	20" - 30"	-	32	-	29
	30" - 40"	-	59	-	-
	40" - 50"	-	16	-	-
	50" +	-	4	-	-

III. Accuracy of existing taper equations

Initial screening of the models was done to detect errors in our formulation of the models and to generate fit statistics for any of the model and data set combinations for which there are published models and species in the data set. This led us to contact David Hann for revisions to the Walters and Hann model. However, as noted above, apparent problems with the published form of the Amidon model, also discovered by Biging in doing his taper study, could not be corrected so only limited comparisons were made with this model.

To illustrate the statistics computed for each model/data set combination, the mean residual and standard deviation for the existing taper models are given for a subset of the data in Tables 4a,

4b, and 4c for Douglas-fir, ponderosa pine, and white fir , respectively[4]. Scanning the residuals for all of the data sets, of which a partial listing is given in Tables 4a to 4c, it became immediately apparent that each of the models performed well (low average residual and low standard error) for some diameter classes within some data sets. However, no model performed uniformly well across all diameter classes for all data sets[5].

In Table 4a for Douglas-fir, it is immediately apparent that the models fall into three groups. Least accurate was the Stadelman model. Second, the "best" model/coefficient combinations are the Biging, Max and Burkhart, Walters and Hann, and Wensel and Krumland model/coefficient combinations. Third, the coast and interior British Columbia models/coefficients by Kozak, Munro, and Smith (1969) for Douglas-fir are not too far off target. This is surprising because their data sources are well outside the region studied here.

In Tables 4b and 4c for ponderosa pine and white fir, respectively, again the Biging, Max and Burkhart, and Walters and Hann models performed the best. No statistics appear here for the Wensel and Krumland model because their model was only fitted to Douglas-fir and redwood on California's north coast. The McTague and Stansfield (1988) model for ponderosa pine also proved to be competitive, even though the model was fitted to data well outside the area of study here.

The Walters and Hann model, while performing well for some data sets, resulted in uniformly larger residual errors. Further, it has several disadvantages for the current application. First, it requires that one know, or estimate, the length of the live crown of the tree. Second, with the estimation of crown length, the model depends on 10 coefficients that must be estimated for each species. Thus, the Walters and Hann model is dropped from further consideration here.

In our present comparison, and in Biging's work with the Max and Burkhart segmented polynomials, the two models, *BIG* and *M&K*, produced similar results. Since Biging's model has fewer fitted coefficients while the Max and Burkhart model has six, the more parsimonious Biging model was selected as the best existing model for all species.

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4 A computer routine, TAPEVAL, was developed to evaluate each of the taper models against any of the data sets shown in tables 2 and 3. The results given in Tables 4a to 4c represent only a subset of the combinations examined by the authors.

5 The Wensel and Olson model, *W&O*, statistics are included in Tables 4a-c for comparison. This is the model that was fit to Wensel and Krumland's model form (1983) here in this study.

Table 4a. Mean bias, residual standard deviation, and sample size by size class and ownership by taper model for Douglas-fir.

Size Class (in)	COOP								Eldorado NF							
	5.5 -18		18 - 30		30 - 40		40+		5.5 -18		18 - 30		30 - 40		40+	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
n	903		478		21				142		296		54		109	
Stadelman	.1	.9	-.3	1.8	-.3	1.6			-.1	1.1	.0	1.9	-.1	2.6	3.7	5.7
Biging	.3	.9	.2	1.3	-.5	.7			.4	.8	.4	1.6	.5	2.3	.5	3.2
M & B	.4	1.0	.3	1.4	-.3	1.0			.6	.9	.7	1.6	-.1	2.3	.9	3.3
W & K	-.2	.9	.0	1.2	.3	.9			.1	.8	.4	1.7	.2	2.6	3.3	5.2
Kozak	.1	1.0	-.3	1.4	-1.1	1.0			.3	.8	.1	1.6	-.8	2.3	.0	3.2
W & H	.6	.9	.7	1.3	1.0	1.1			.6	.9	1.0	1.7	.7	2.5	2.9	4.6
W & O	.2	.9	.4	1.3	.2	.8			.5	.7	.6	1.6	.3	2.4	2.6	3.7
	Michigan-Cal								Klamath NF							
n	132		514		164		103		291		319		144		649	
Stadelman	-.1	1.4	-.4	1.9	-.1	2.4	-.6	4.0	.2	1.0	-.4	1.5	-.8	2.6	.2	2.9
Biging	.8	1.0	.3	1.6	-.1	1.9	-1.7	3.4	.3	.7	-.1	1.1	-.8	2.3	-2.0	3.0
M & B	.9	1.0	.6	1.6	.3	1.9	-1.2	3.3	.5	.7	.1	1.1	-.4	2.3	-1.4	2.9
W & K	.4	.8	.4	1.6	1.0	1.9	1.7	3.5	-.1	.6	-.2	1.1	-.3	2.4	1.2	3.8
Kozak	.6	.9	.1	1.6	-.5	1.9	-2.5	3.4	.2	.7	-.4	1.1	-1.1	2.4	-2.7	3.3
W & H	.3	1.0	.1	1.9	.1	2.2	-1.2	4.5	.5	.8	.6	1.2	.0	2.7	-1.8	5.0
W & O	.6	.9	.3	1.6	.4	1.8	-.3	3.2	.4	.7	.4	1.2	.2	2.4	-.6	3.0
	Bulletin 1883								Six Rivers							
n	205		253		164		70		74		94		14		33	
Stadelman	.5	1.0	-.4	1.4	.0	1.9	.9	3.2	-.1	1.1	-.0	2.1	-1.6	2.0	-.5	3.5
Biging	.1	.8	-.2	1.2	-.3	1.4	-.6	2.1	.7	.7	.6	1.7	-1.6	2.2	-2.3	3.9
M & B	.3	.8	.1	1.2	.1	1.4	-.1	2.1	.8	.8	.9	1.8	-1.2	2.0	-1.7	3.7
W & K	-.3	.8	-.2	1.2	.6	1.8	1.4	2.8	.3	.6	.7	1.8	-.7	2.1	1.0	3.1
Kozak	-.1	.8	-.4	1.2	-.6	1.4	-1.1	2.1	.5	.7	.3	1.7	-2.0	2.3	-2.9	4.1
W & H	.6	.8	1.0	1.3	1.7	1.9	2.1	3.0	.7	.9	1.2	2.0	.1	2.4	.3	3.9
W & O	.1	.8	.1	1.1	.4	1.3	.7	1.9	.5	.7	.6	1.7	-1.2	2.0	-.8	3.3

Table 4b. Mean bias, residual standard deviation, and sample size by size class and ownership by taper model for Ponderosa pine.

Size Class (in)	COOP								Eldorado NF							
	5.5 -18		18 - 30		30 - 40		40+		5.5 -18		18 - 30		30 - 40		40+	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
n	734		892		61				317		495		214		114	
Stadelman	.0	1.1	-.2	1.5	-.3	2.7			-.2	1.4	-.7	1.9	-.2	2.4	-.0	4.4
Biging	.3	.7	.5	1.1	.7	2.2			.2	.7	.5	1.3	.5	2.0	2.0	2.9
M & B	.4	.7	.5	1.0	.6	2.2			.4	.7	.6	1.3	.6	1.9	2.0	2.8
Kozak	-.4	.7	-.7	1.2	-1.1	2.2			-.3	.7	-.5	1.3	-.9	2.0	.2	2.7
W & H	.7	.8	1.0	1.3	.9	2.3			.3	.9	1.0	1.5	1.0	2.4	2.5	3.7
W & O	.4	.7	.4	.9	.3	2.0			.1	.8	.2	1.3	-.1	1.9	1.0	2.7
	Michigan-Cal								Klamath NF							
n	207		1007		439		82		389		459		54			
Stadelman	-.4	1.8	-.5	1.7	-.9	2.3	-2.3	2.9	-.5	1.1	-2.1	1.5	-1.1	1.5		
Biging	.4	1.2	.3	1.2	.2	2.0	.0	2.2	-.1	.7	-.2	1.2	-.0	1.1		
M & B	.3	1.2	.2	1.2	.1	1.9	-.0	2.3	.0	.7	.2	1.4	.6	1.1		
Kozak	-.1	1.2	-.7	1.2	-1.2	2.0	-2.0	2.3	-.6	.8	-1.0	1.3	-1.3	1.3		
W & H	.0	1.4	.1	1.6	.2	2.3	.1	2.7	.3	.7	.5	1.3	.9	1.7		
W & O	.2	1.3	.1	1.2	-.2	1.9	-.9	2.3	-.3	.8	.3	1.3	.1	1.4		
	Bulletin 1883															
n	150		211		102		33									
Stadelman	-.5	1.4	-1.1	2.0	.2	2.4	-.6	4.1								
Biging	-.1	1.0	.0	1.4	1.0	1.7	1.4	3.0								
M & B	.0	1.0	.2	1.3	1.1	1.6	1.6	2.8								
Kozak	-.8	1.1	-1.0	1.5	-.5	1.6	-.7	2.9								
W & H	.5	1.0	1.1	1.6	1.9	2.3	3.3	4.0								
W & O	-.3	.9	-.2	1.4	.5	1.5	.4	2.9								

Table 4c. Mean bias, residual standard deviation, and sample size by size class and ownership by taper model for white fir.

Size Class (in)	COOP								Eldorado NF							
	5.5 -18		18 - 30		30 - 40		40+		5.5 -18		18 - 30		30 - 40		40+	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
n	1343		894		39				351		515		199		111	
Stadelman	-.5	1.1	-.3	2.0	-1.1	2.1			-.9	1.3	.1	2.2	-.3	2.8	.9	4.3
Biging	.3	.9	-.0	1.4	-1.0	1.3			.3	.9	.3	1.6	.3	2.1	-.1	3.9
M & B	.4	.8	.1	1.2	-.8	.9			.2	.9	.2	1.6	.2	2.3	-.3	4.1
W & H	.7	.9	.9	1.4	.7	1.2			.6	1.1	1.3	1.9	2.3	2.9	3.1	5.3
W & O	.5	.7	.5	1.3	.1	1.4			.3	1.3	.6	1.5	1.0	2.2	1.3	4.0
	Michigan-Cal								Klamath NF							
n	719		1991		560		83		221		212		106			
Stadelman	-.4	1.6	-.5	2.0	-1.3	2.3	-1.5	7.7	-.8	1.1	-.9	2.0	-2.1	2.0		
Biging	.6	1.0	.2	1.3	-.7	1.9	-1.2	8.2	.2	.8	-.5	1.7	-1.2	2.0		
M & B	.5	1.0	.2	1.4	-.5	1.9	-1.2	8.2	.2	.8	-.3	1.8	-.8	2.1		
W & H	.4	1.1	.4	1.6	.3	2.2	.6	8.8	.4	.9	.5	1.8	.3	2.1		
W & O	.6	1.0	.3	1.3	-.1	1.8	-.3	8.3	.3	.8	.0	1.7	.0	2.1		
	Bulletin 1883								Six Rivers							
n	128		313		125		6		125		168		129			
Stadelman	-1.2	1.2	-.6	1.7	-.6	2.3	2.2	3.5	-.5	1.4	-.0	2.2	-.1	2.8		
Biging	-.5	1.1	-.5	1.4	-.4	2.0	2.2	2.7	.5	1.0	.1	1.7	.2	2.4		
M & B	-.4	1.0	-.4	1.3	-.2	2.0	2.5	3.6	.5	1.0	.1	1.6	.1	2.4		
W & H	.6	1.0	1.1	1.6	2.1	2.6	7.4	5.4	.7	1.1	1.3	1.7	1.8	2.8		
W & O	-.2	1.0	.0	1.2	.6	1.7	3.6	3.7	.6	.9	.5	1.5	.9	2.2		

IV. Refitting Coefficients

Based upon the the performance of the models with the coefficients fitted by the authors, both the *BIG* and *W&K* models were selected for refitting here. A composite data set was divided into two subsets (every other tree) for each of the major species, one half for fitting and the other for testing. Bark thickness was predicted, where necessary, using the model developed by Wensel and Olson (1991). Wensel and Olson did not provide bark thickness models for Jeffrey pine and lodgepole pine, so the model for ponderosa pine was used for these species. The fit data set used, described in Table 5, was developed by combining all of the tree data available and then screening the data for completeness. As shown in Tables 5 and 6, this resulted in the tree taper measurements for fitting on over eight thousand trees yielding over 28 thousand individual upper stem taper measurements ($\geq$ 4.5 ft.).

Data Screening and Editing

The data were received from the various contributors and cooperators in this project and stored in a common data base. The data were then retrieved from this data base using a screening

process that eliminated trees and/or observations that were obviously in error or would result from grossly misformed trees. The data were screened to eliminate trees that had forked or broken tops. Since field data sheets for most of the data were not readily available, data were not edited or corrected. Since measurement error can have the effect of moving an observation closer or further from the mean, outliers were not deleted as this would result in one-way bias.

Table 5. Number of trees, mean, minimum, and maximum diameter at breast height and total height, and number of observations below and at or above breast height used to fit Wensel and Krumland and Biging stem taper models by species.

		Diameter			Height			Number of observations	
Species	Number of trees	Mean	Min	Max	Mean	Min	Max	height <4.5 ft	≥4.5 ft
Douglas-fir	995	22.5	5.5	87.9	106.6	25.5	265.0	902	5249
Ponderosa pine	1907	21.5	5.1	94.5	91.0	18.0	238.4	2038	8094
Jeffrey pine	945	30.8	5.2	60.9	83.9	14.0	175.8	1049	4951
Sugar pine	583	24.9	6.0	69.2	99.5	18.1	202.0	606	2487
Lodgepole pine	215	16.4	7.9	44.4	66.9	27.7	147.9	320	940
White fir	2384	21.1	5.2	75.7	91.4	18.9	249.5	2505	9640
Red fir	911	23.2	5.5	65.7	88.7	20.6	207.2	956	4432
Incense cedar	645	22.5	5.2	73.0	73.0	10.7	211.6	666	2718

Model Development

The modified Gauss-Newton method of nonlinear model parameter estimation was used to estimate coefficients for the various nonlinear models considered. Projecting on to the solution surface for Wensel and Krumland's model, being a five parameter model, required searching a grid to ensure that the solution was indeed the global minimum residual sum of squares. Especially for white fir, there were several local minima quite close to the global minimum.

There was considerable correlation between coefficients c_2, c_3, and c_4 as these were the coefficients that adjusted for the height-diameter relationship. However, for Douglas-fir, ponderosa pine, Jeffrey pine, and lodgepole pine, the c_4 coefficients was found to be not significant in the model. In these species, the coefficient for total height (c_4) was set at zero resulting in the other four coefficients becoming significant. For lodgepole pine, the coefficients for both DBH and total height (c_3 and c_4) were zero resulting in a model very similar to Biging's.

Coefficients for the Commercially Important Conifer Species

Upper stem - Tables 6 and 7 show the coefficients calculated using models developed by Biging and Wensel and Krumland for the eight species that are both commercially important in California and for which there are sufficient numbers of observations for fitting. Table 6 gives the coefficients and mean squared error (MSE), corrected for degrees of freedom, calculated for Biging's model. The MSE ranged from a low, for lodgepole pine, of 1.78 sq. in. to 3.21 sq. in. for sugar pine. Table 7 gives the coefficients calculated for Wensel and Krumland's model along with MSE. The MSE ranged from 2.91 sq. in. for sugar pine to 1.45 sq. in. for white fir.

Table 6. Coefficients, mean square error (MSE), and number of observations (n) for fit of stem taper data, breast height and above, collected from throughout California to the model developed by Biging (1986).

Species	n	b_1	b_2	MSE
Douglas fir	5249	0.99007	0.32640	3.05
Ponderosa Pine	8094	1.04451	0.33229	1.98
Jeffrey Pine**	4951	1.00691	0.30606	3.74
Sugar Pine	2487	1.06171	0.37902	3.21
Lodgepole Pine**	940	1.17304	0.45390	1.78
White fir	9640	1.04442	0.33571	1.87
Red fir	4432	1.01513	0.31803	1.85
Incense cedar	2718	1.00459*	0.40012	2.23

* Not significantly different from 1.0 at $\alpha = 0.05$
** Ponderosa pine bark model was used for converting outisde-bark observations.

A comparison of the residual error plots for the Biging and Wensel and Krumland models can be made by referring to Figures 1 to 4. Comparing the residuals for Douglas-fir, Figures 1 and 3, and for ponderosa pine, Figures 2 and 4, reveals no significant bias in the predictions. The same was true for the other species.

Table 7. Number of observations (n), coefficients, and mean square error (MSE) for fit of stem taper data, breast height and above, collected from throughout California to the model (equation 1) developed by Wensel and Krumland (1983).

Species	n	c_0	c_1	c_2	c_3	c_4	MSE
Douglas-fir	5249	0.84292	.97062	-0.38163	-0.0074002	0.0	2.52
Ponderosa Pine	8094	0.87278	1.26066	-1.91214	0.020445	0.0	1.54
Jeffrey Pine*	4951	0.82932	1.50831	-4.08016	0.047053	0.0	2.95
Sugar Pine	2487	0.90051	0.91588	-0.92964	0.0077119	-0.0011019	2.91
Lodgepole Pine*	940	1.0	0.84257	-0.98434	0.0	0.0	1.73
White fir	9640	0.86039	1.45196	-2.42273	-0.15848	0.036947	1.45
Red fir	4432	0.87927	0.91350	-0.56617	-0.014480	0.0037262	1.52
Incense cedar	2718	1.0	0.31550	-0.34316	0.0	-0.00039283	2.16

* Ponderosa pine bark model was used for converting outisde-bark observations.

To test and compare the *W&O* and *BIG* models the mean residual and the mean squared error was calculated using the observations withheld from the fitting data set (Table 8). Again, the W&O model had consistently smaller MSE and, except for white fir, consistently smaller mean residual.

Table 8. Mean residual and mean squared error for test of *W&O* and *BIG* models on test data by species.

		W&O		*BIG*	
Species	n	Mean Residual	Mean Squared Error	Mean Residual	Mean Squared Error
Douglas-fir	5253	-.051	2.99	.080	3.49
Ponderosa pine	8023	-.078	1.62	-.121	2.04
Jeffrey pine	4939	-.012	2.85	-.027	3.59
Sugar pine	2533	-.056	3.06	-.126	3.26
Lodgepole pine	988	.020	1.65	.026	1.69
White fir	9692	-.059	1.51	-.019	1.87
Red fir	4573	.065	1.69	.137	1.94
Incense-cedar	2740	-.016	2.40	.070	2.45

The correlation between residual error and diameter at breast height was consistently low using the *W&K* model (< 10%) while the correlation using the *BIG* model was consistently higher (Table 9)[6]. For the W&K model, these correlations are not high enough to be important. Also, for several of the species shown in Table 9, the same can be said for the Biging model.

Table 9. Correlation between residual and DBH for models fit for the *BIG* and *W&K* models.

	W&K	*BIG*
Douglas-fir	-0.09	-0.28
Ponderosa pine	0.09	0.15
Jeffrey pine	0.10	0.12
Lodgepole pine	-0.09	-0.12
Sugar pine	-0.04	0.07
White fir	-0.03	-0.12
Red fir	-0.09	-0.24
Incense-cedar	-0.02	-0.12

Stump Model - The stump model proposed by Wensel and Krumland (1983) produced results with minimal bias and the least overall mean squared error. Although a linear model with a intercept term, using DBH and height as independent variables, was nearly as good at predicting stump diameters (less than 5 percent increase in MSE), such a model is conceptually inappropriate because of the intercept term .

The coefficients and mean squared error computed for Wensel and Krumland's stump model are shown in Table 10. Jeffrey pine and incense-cedar had MSE's in excess of six sq. in. while Lodgepole pine had a MSE of 1.51 sq. in. The error in predicting stump diameters was much greater than for predicting upper stem diameters. This is to be expected, as the magnitude of the observations was much larger, and there is greater variation in taper in the lower stem. Neither of the residual plots for the Wensel and Krumland stump model for Douglas-fir or ponderosa pine, Figures 5 and 6, respectively, show any bias in the predictions. The same was true for the other species.

[6] A 3-parameter version of Biging's model was developed for each species (with m estimated instead of being equal to $\frac{1}{3}$). Compared with the 2-parameter version, this had only slight reductions in the overall residual sums of squares and very little or no reduction in the correlations shown in Table 8. Thus, as discovered by Biging, there is little reason to prefer the 3-parameter version of this model over the 2-parameter version.

Table 10. Sample size (n) and computed coefficients and mean squared error (MSE) for Wensel and Krumland's stump model (HT $\leq$ 4.5).

	n	b_0	b_1	MSE
Douglas-fir	902	.1420	.04302	4.11
Ponderosa pine	2038	.1031	.03068	2.50
Jeffrey pine	1049	.1472	.03880	6.20
Sugar pine	606	.0743	.02936	3.91
Lodgepole pine	320	.0147	.03223	1.51
White fir	2505	.0844	.03320	3.25
Red fir	956	.1105	.05061	4.37
Incense-cedar	666	.1177	.03894	6.33

V. Influence of site and location parameters

It is common practice to produce "localized" versions of tree volume and taper models because of perceived differences in the predicted relationships as one changes location. First, the effect of latitude and elevation are examined followed by an analysis of the effect of site quality.

Latitude and Elevation

One of the objectives of this study was to adjust taper predictions for each species for changes in geographic location and site quality, if appropriate. California stretches over 9 degrees of latitude; we have data from the San Bernardino National Forest at 34 degrees to the data from Oregon at 43 degrees. Because of this, latitude was used as a measure of relative geographic position. Other approaches to assessing the effect of geographic location on stem taper could have been investigated, such as nonparametric analyses based on the concept of local genetic variation, but these approaches were not possible with the existing information.

Trees in the database were assigned a latitude based upon either specific plot information or, if plot information was not specific with respect to latitude, a latitude was assigned approximating the geographic center of the ownership or National Forest that the tree came from. These approximated or "pseudo" latitudes were then used to determine the effect of latitude and geographic position on stem taper.

Elevation information was less readily available for trees in the database. No attempt was made to approximate elevations if they were absent from the plot data. For assessing the effect of elevation on stem taper, elevation was corrected for latitude. It is generally known that with increasing latitude there is a lowering of equivalent climatic zones. However, there is little reported in the literature as to the specific altitudinal change per unit of latitudinal change for California.

Daubenmire (1954) reported that timberline drops 360 feet per degree of latitude increase under a given type of climate regime for the Rocky, Coastal, and Appalachian Mountains. In the absence of a more definitive relationship, Daubenmire's 360 feet per degree of latitude relationship was used to adjust for latitudinal differences in observations.

Early attempts to correlate residuals from predicted diameters with latitude used the taper models reported by Biging (1984) and Wensel and Krumland (1983). For this, actual latitude, pseudo-latitude, and latitude corrected for elevation were used. linear correlations for the six major conifers are shown in Table 11. An examination of the residual plots showed no correlation, either linear or non-linear, of the residuals with latitude and/or elevation. As a result, no local geographic adjustment was made to the taper models for each species.

Table 11. Correlation between latitude and elevation versus residual (observed minus predicted inside bark diameter) and sample size (in parentheses) using Wensel & Krumland's (1983) and Biging's (1984) stem models to predict inside bark diameters.

Model	--------Wensel & Krumland--------				----------Biging----------			
Species	L	PL	LT	LE	L	PL	LT	LE
DF	.002	-.044	.002	-.007	.033	-.063	.033	-.031
	(1854)	(317)	(1854)	(1854)	(1854)	(317)	(1854)	(1854)
PP	.107	-.131	.107	-.075	.059	-.116	.059	-.049
	(2920)	(1048)	(2920)	(2920)	(2920)	(1048)	(2920)	(2920)
SP	.147	.063	.147	-.143	.054	.032	.054	-.073
	(1063)	(436)	(1063)	(1063)	(1063)	(436)	(1063)	(1063)
WF	.037	.022	.040	-.043	.041	.045	.041	-.055
	(6290	(3731)	(6361)	(6361)	(6290	(3731)	(6361)	(6361)
RF	.055	-.001	.035	-.051	.049	-.009	.030	-.053
	(4631)	(4680)	(5637)	(5637)	(4631)	(4680)	(5637)	(5637)
IC	.127	.081	.127	-.142	.023	.062	.023	-.008
	(1667)	(1158)	(1667)	(1667)	(1667)	(1158)	(1667)	(1667)

L = Latitude

PL = Pseudo latitude based upon center of ownership or National Forest

LT = Total Latitude, i.e, latitude, if present, or pseudo-latitude.

LE = Latitude corrected Elevation: LE = (Elev - 4000') + (Latitude - 39°) * 360 (ft/degree)

Site quality

To see if there was any effect of site quality on the taper of trees an evaluation of site versus residuals was made. All observations for which there was 50-year site information were used to evaluate site versus residuals, and the correlations are given in Table 12. The correlation between site and residual ranged from -0.15 for incense-cedar to 0.03 in ponderosa pine for the two Biging and Wensel and Krumland models. These correlations, by inspection, do not appear high enough to warrant the further investigation of site index adjustments to the models. This is particularly true considering the wide range of site qualities present in the data. One cannot say, however, that the trees have the same taper regardless of site quality because the Wensel and Krumland model varies the taper by tree DBH and height, which will themselves vary by site quality.

Table 12. Correlation between site index and residual of predicted versus observed inside bark diameters, sample size, and range of site indexes by species.

Species	n	50-year Site Range	------Correlation Site vs. residual------ Wensel & Krumland	Biging
Douglas-fir	223	39-111	.05	-.01
Ponderosa pine	775	22-124	.02	.03
Sugar pine	348	26-116	-.13	-.11
White fir	2122	28-132	.05	.02
Red fir	3361	23-110	.01	-.03
Incense-cedar	860	19-119	-.07	-.15

VI. DISCUSSION

Taper equations are used for appraisal work by estimating the scaling diameters of logs in the upper stem of trees. These diameters are used to get the scaling volume of trees which, when coupled with defect and log grade, are used to estimate the value of each tree. Here we have attempted to assess the reliability of various taper models that are currently being used for this purpose in California and to estimate the coefficients for several of the taper models.

New coefficients were fitted to a model developed by Wensel and Krumland for Douglas-fir and redwood (*Sequoiadendron sempervirons* (D.Don) Endl.) stem taper for the more important commercial conifer species in California. The coefficients were fit to stem taper data collected throughout California as part of numerous previous individual studies of stem taper involving both felled and standing tree observations. The newly developed coefficients perform generally better

throughout California on an overall basis, having generally less bias and variance compared to previously used models. However, other model forms may do as well or better for some species. For example, newly fitted coefficients for Kozak's model (Kozak 1988) produced a significant reduction in the residual sum of squares for Douglas-fir, but showed little or no improvement for the other species tested. Also, previously developed models perform well in some diameter size classes for certain data sets. The Wensel and Krumland model is a five parameter model using relative height, diameter at breast height, and total height as predictive variables. The ability of this model to adjust for a larger range of diameters and heights appears to be the reason for its more general applicability over the 9 degrees of latitude from southern California to central Oregon.

The Wensel and Krumland model has two predictive equations, one for breast height and above and one for breast height and below. The below breast height model is provided for those who wish to predict cubic volume, including the flare below breast height.

Latitude, elevation, and site quality as measured by site index were evaluated to see if there was any need to develop either local models or include these variables into the models. There was little or no relationship between model residual error and latitude corrected for elevation or between residual error and site. Thus, the same coefficients can be used throughout the region.

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Residual Plots -

Figure 1. Residuals versus predicted for Douglas-fir using Wensel and Krumland's model.

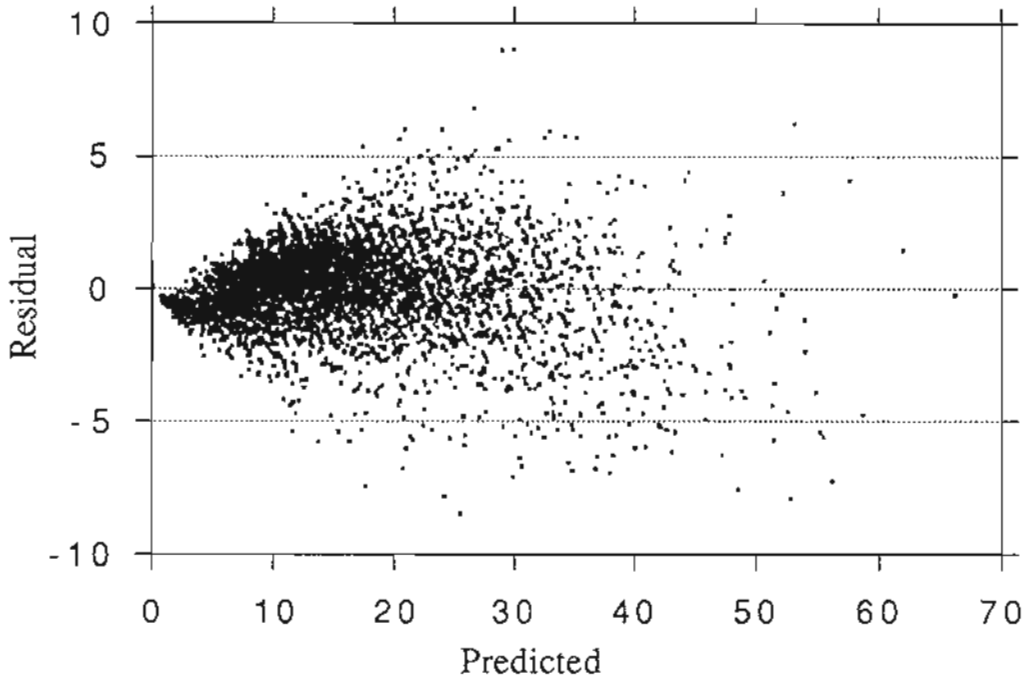

Figure 2. Residuals versus predicted for ponderosa pine using Wensel and Krumland's model.

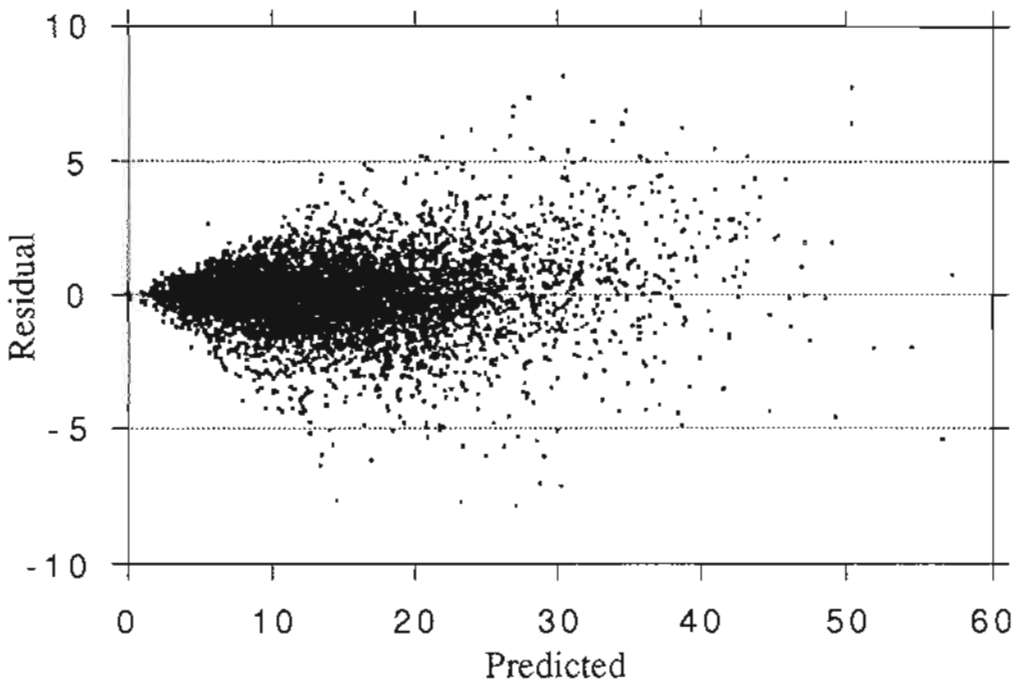

Figure 3. Residuals versus predicted for Douglas-fir using Biging's model.

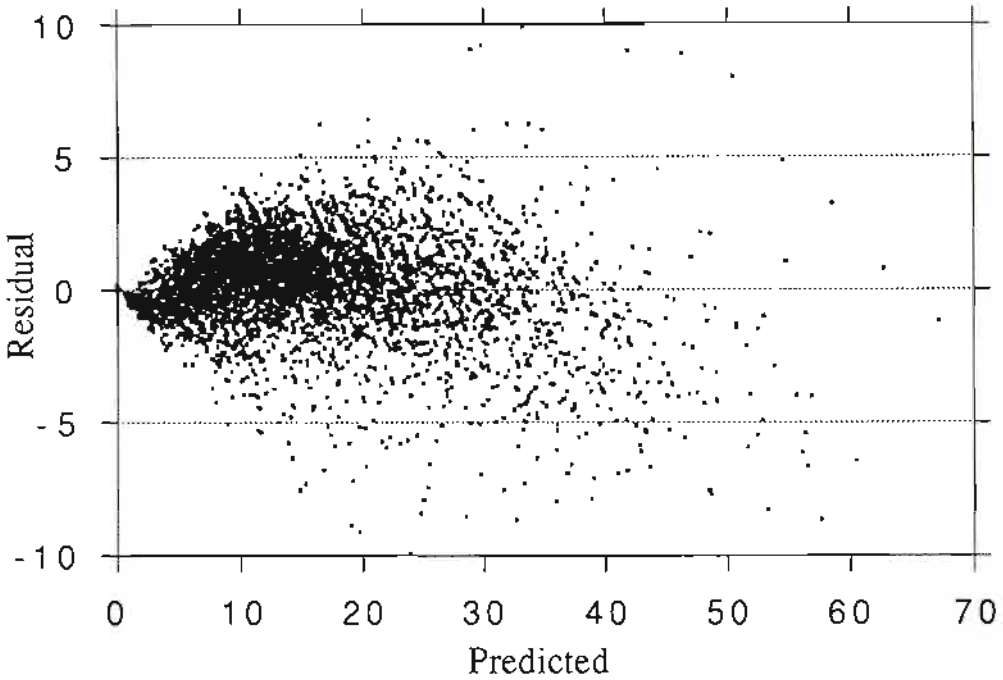

Figure 4. Residuals versus predicted for ponderosa pine using Biging's model.

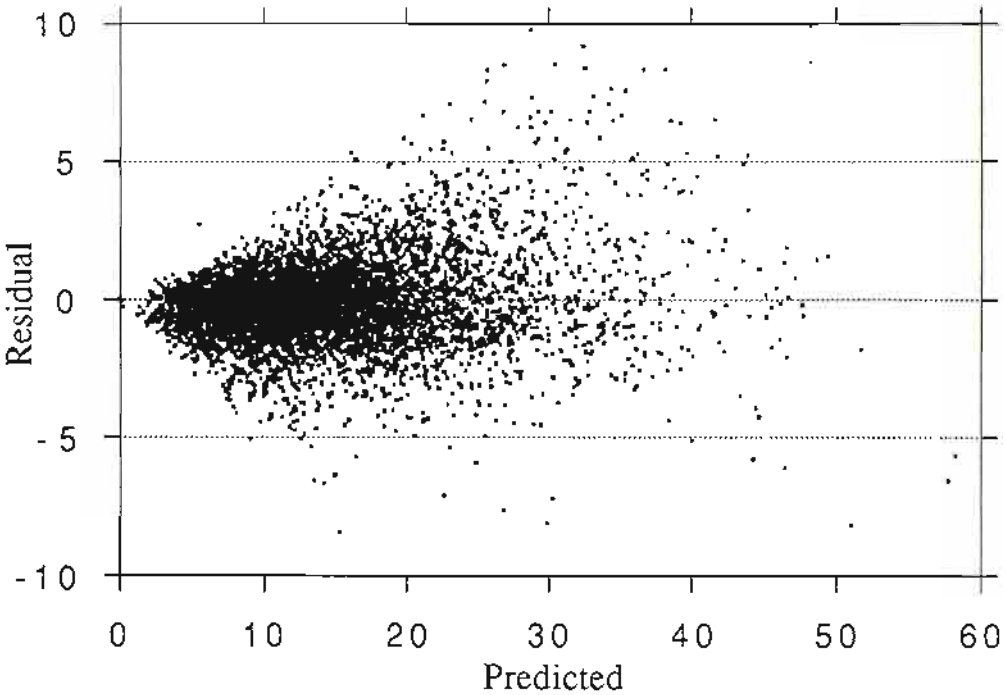

Figure 5. Residuals versus predicted for Douglas-fir using Wensel and Krumland's stump model (ht < 4.5 ft.).

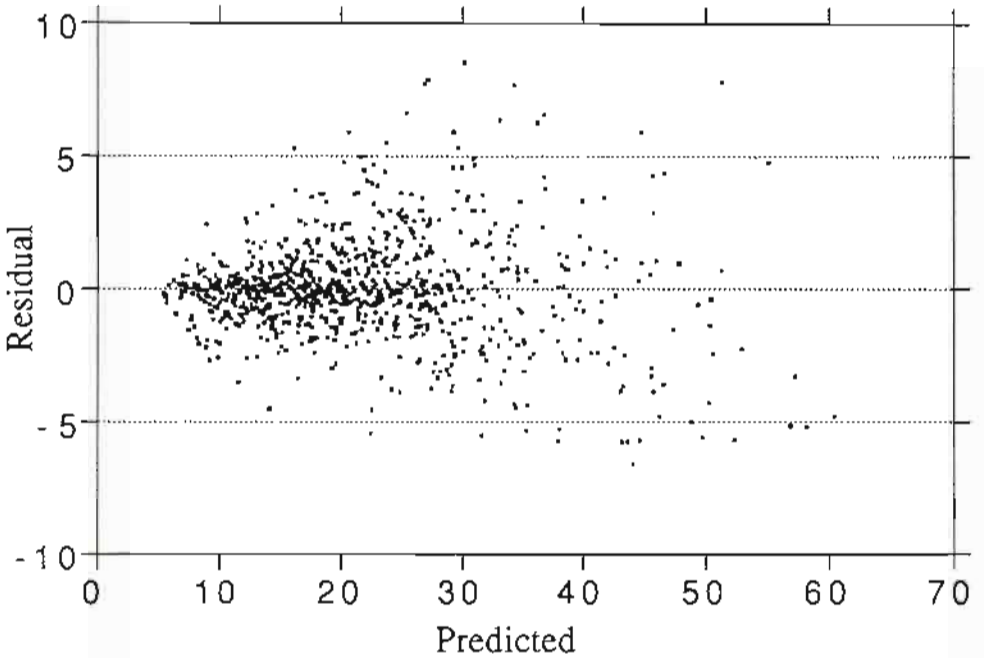
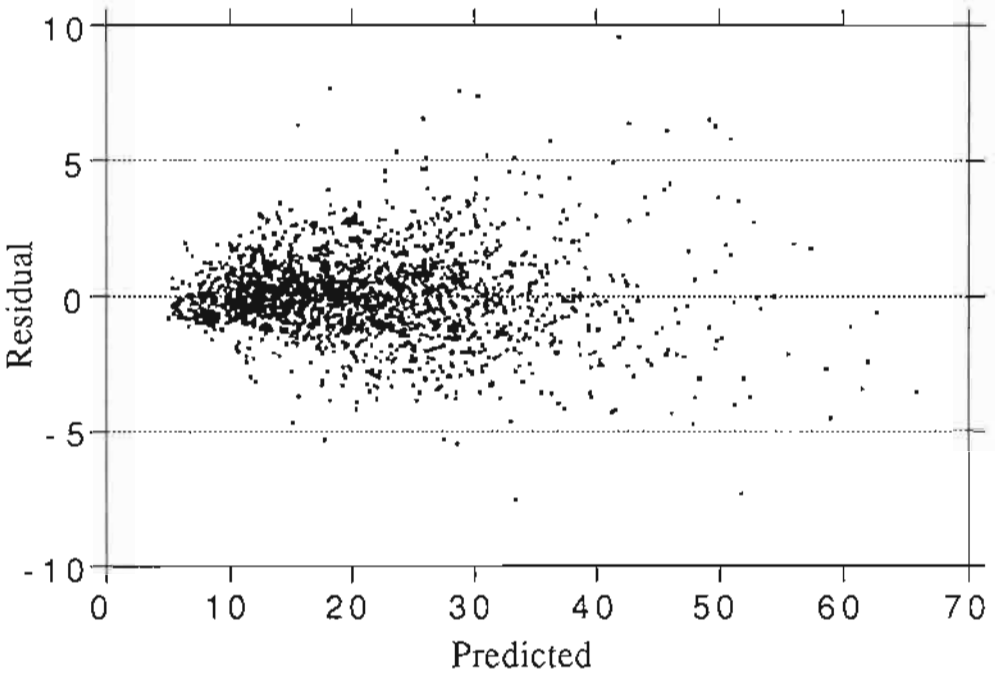
Figure 6. Residuals versus predicted for ponderosa pine using Wensel and Krumland's stump model (ht < 4.5 ft.).