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REVISED DRAFT

Volume Tables for Young-growth Mixed Conifers of Northern California based upon the Stem Analysis Data

Part I: Volume Tables for Mixed Conifers

Part II: Taper Equations for Mixed Conifers

by

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Abstract

This paper presents a taper equation developed to describe stem form for young-growth mixed conifers in Northern California based upon stem analysis data collected by the Northern California Forest Yield Cooperation (NCYFC). These models can be used to predict upper stem diameters as a function of relative height and DBH. Additionally, these models can be used to create volume tables to any merchantable top diameter.

This note is comprised of two parts. The first section provides an overview of the project and presents the volume tables arising from the study. The second section is a technical report describing the development of a taper equation and accuracy of the model.

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Definition of Variables

Variable(s)	Definition
b_1, b_2	Regression coefficients for the taper equation
c_0, c_1, c_2	Regression coefficients for the tree volume equation
d	Predicted diameter inside bark (in.)
dib	Diameter inside bark (in.)
DOB	Diameter outside bark (in.)
DBH	Diameter outside bark at breast height (in.)
h	Distance in feet above ground to some point on the tree bole.
H	Total height
y=h/H	Relative height above ground
x=d/DBH	Relative diameter
λ	A constant in the taper equation which forces the equation to go through the tip of the tree.
ln(x)	Natural logarithm of x
exp(x)	2.7183 raised to the power of x
$S_{y.x}$	Standard deviation of the residuals about the regression of "y" on "x"
R^2	Squared multiple correlation coefficient
1/m	allometric constant

I. Introduction

This report describes the development of taper equations for the mixed conifer species of Northern California. These species include ponderosa pine (PP), Douglas fir (DF), white fir (WF), red fir (RF), sugar pine (SP) and incense cedar (IC).

Data from over 1,000 stem analysis trees felled by the NCFYC Coop between 1980 and 1981 were utilized to characterize stem profile. A subset of the data was selected to develop a flexible taper equation that accurately describes stem form while another portion was retained for testing purposes. These equations were then used as a basis for generating cubic foot and board foot volume tables. In all cases a 1-foot stump height is assumed.

General taper equations may be integrated mathematically to produce tree volume tables. However, this approach will not lead to volumes identical to those predicted from tree volume prediction equations. This is because the volume of the treee of average profile for a given height and diameter class is not necessarily equal to the average volume of the trees in that class. Krumland and Wensel(1983) compared these two approaches in their work with redwood (Sequoia sempervirens) and found that the taper-based volume and tree volume predictions were generally within 1-3 per cent of each other when predicting cubic volume to a 5-inch top. For Douglas fir (Psuedotsuga mensiesii) the two approaches led to volume differences within 5 per cent of each other. This difference notwithstanding, taper equations provide the means to assess upper stem diameters that are requisite for making deductions by log position. Too, log product mix can be predicted via taper equations whereas volume equations only estimate whole tree volumes. It is also possible to use taper equations to apportion the volumes calculated from whole tree volume equations.

II. Summary of Results

A. Taper Equations

The taper equations developed to describe stem form of young-growth mixed conifers in northern California based upon the NCFYC Coop Stem Analysis data was:

$$d = DBH[b_1 + b_2 \ln (1-\lambda(h/H)^{1/3})] \quad \text{where } \lambda=1-\exp(-b_1/b_2)$$

The coefficients and fit statistics are given in Table 1 for the six species considered.

Table 1. Coefficients and fit statistics for the taper equation

	Species					
	PP	DF	RF	WF	SP	IC
b_1	1.019589	1.029288	1.075880	1.092615	1.06932	1.071343
b_2	0.335666	0.334012	0.353784	0.365295	0.415632	0.472157
n	2014	1588	312	2645	692	541
number of trees	254	202	43	361	84	95
R^2	0.9907	0.9895	0.9911	0.9895	0.9920	0.9894
$S_{y.x}$ (in.)	1.2436	1.0704	1.1069	1.1838	1.1535	1.0213

R^2 = Squared multiple correlation coefficient for d.

$S_{y.x}$ = Standard deviation of the residuals about the regression of y on x.

This model can be used to obtain tree volumes by section as well as whole tree volumes. The volume tables presented herein were generated with these equations.

B. Volume Equations

An alternative to taper-based volume estimates is the development of volume equations for estimating whole tree volume. These models can

not be used to predict volume by section, but should provide slightly better estimates of whole tree volume. The form of these models was:

$$\ln(V) = c_o + c_1 \ln(DBH) + c_2 \ln(H)$$

where ln = natural logarithm

The tree volume coefficients and fit statistics are given in Table 2 for the six species considered.

Table 2. Fit Statistics and Coefficients for the Volume Equation

$$\ln(V) = c_0 + c_1 \ln(DBH) + c_2 \ln(H)$$

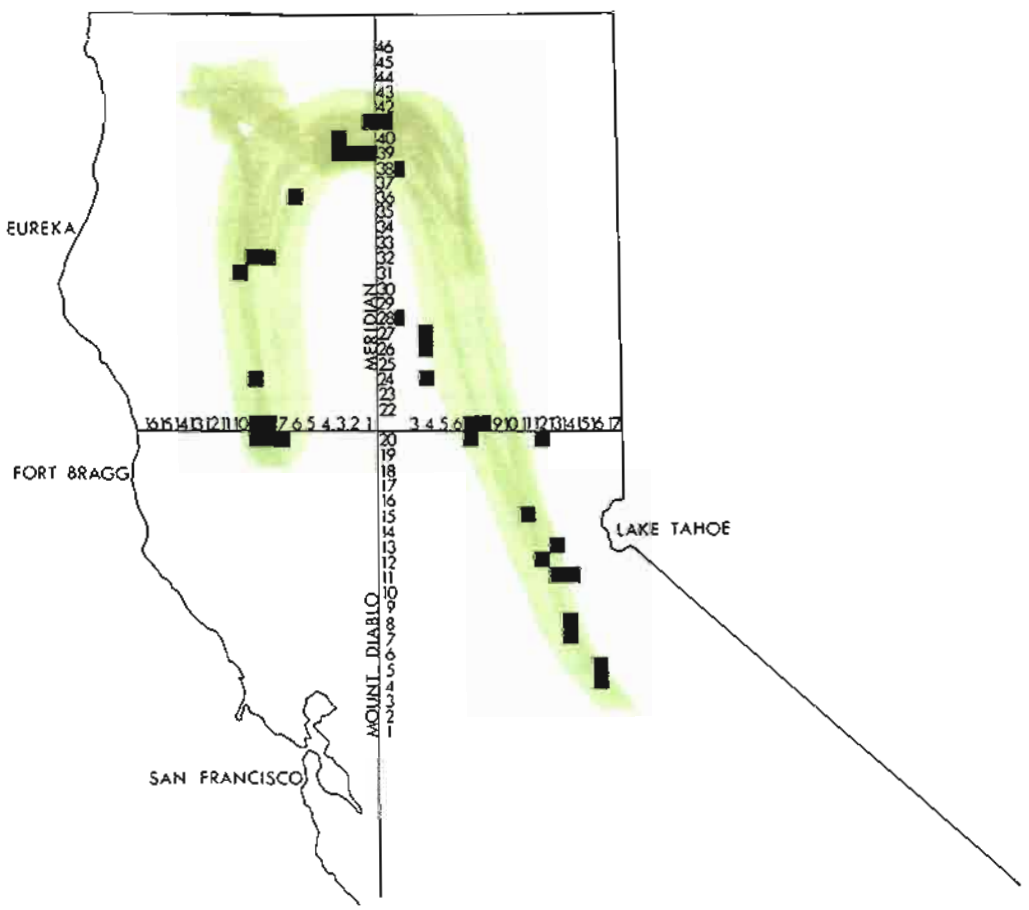
		Species					
		PP	DF	RF	WF	SP	IC
Cubic foot 4" top	c_0	-5.87037	-6.77371	-7.42110	-6.58856	-6.30458	-6.20820
	c_1	2.07856	1.74091	1.69539	1.74371	1.89487	1.86583
	c_2	.84577	1.27009	1.46867	1.25399	1.05278	1.02396
	n	239	165	42	306	80	75
	R^2	.97	.97	.95	.98	.98	.97
	Sy.x (cu ft)	9.17	5.99	7.35	5.85	6.90	3.21
Cubic foot 6" top	c_0	-6.27940	-7.31263	-8.19624	-7.07110	-6.64006	-6.85535
	c_1	2.27130	1.86620	1.74634	1.82530	1.98574	2.09022
	c_2	.80214	1.30151	1.60370	1.30290	1.06021	1.01111
	n	239	165	42	306	80	75
	R^2	.96	.97	.95	.97	.98	.96
	Sy.x (cu ft)	10.42	6.56	7.50	6.28	7.05	3.67
Board foot 6" top	c_0	-6.69958	-7.56070	-9.65279	-7.71557	-7.31125	-8.49627
	c_1	2.46342	2.01373	1.86991	1.94935	2.17057	2.09810
	c_2	1.13365	1.62982	2.22310	1.73931	1.44848	1.73953
	n	239	165	42	306	80	75
	R^2	.96	.96	.95	.97	.97	.93
	Sy.x (bd ft)	69.04	41.05	49.13	43.02	47.11	24.04
Board foot 8" top	c_0	-7.31642	-9.00895	-10.74943	-8.47169	-9.05036	-8.55107
	c_1	2.86479	2.45348	2.15186	2.26747	2.51420	2.77447
	c_2	.99215	1.65063	2.27388	1.69054	1.58614	1.26352
	n	229	157	42	300	80	66
	R^2	.93	.94	.94	.95	.97	.93
	Sy.x	94.35	58.53	54.45	52.22	52.69	25.34

III. Data Sources

Data for this study were provided by the Northern California Forest Yield Cooperative growth and yield project. This study, in the mixed conifer region of California, combines efforts of twelve private industries and the University of California, Berkeley. In the stem analysis portion of the Coop study, thirty clusters containing 3 one-fifth acre (.08 ha) plots and eight clusters containing two one-tenth acres (.04 ha) plots were located in northern California (Figure 1).

Every plot was stem mapped and DBH, total height and crown length were recorded for trees greater than 6 inches (15.2 cm) in diameter at breast height (DBH). On each plot, approximately one dozen trees were felled. Four to six dominants (two to three for each of the two most prevalent species in the overstory) were chosen randomly for felling from the plot as site index trees. Up to seven additional trees were felled for stem analysis on each plot and were randomly selected, but proportional to their representation in the following diameter classes: 6-10.99"; 11-12.99"; 13-14.99"; 15-16.99"; 17-18.99"; 19-20.99" and >20.99" which corresponds to 15.2-27.9 cm; 27.9-33.0 cm; 33.0-38.1 cm; 38.1-43.2 cm; 43.2-48.2 cm; 48.3-53.3 cm and >53.3 cm. No more than four trees were selected from any of these diameter classes. This method ensured that trees would be selected from all size classes, but in approximate proportion to their occurrence on the plot.

Figure 1. Location of stem analysis plots by township



Each tree selected for stem analysis was felled and section rounds (1-2in (2.5-5.1 cm) thick) were taken at stump height (1.5 ft (0.46 m)), breast height and subsequent log lengths (16.5 ft (5.03 m) or 20.5 feet (6.25 m)). Each section was tagged and photographs taken. Laboratory analysis to determine age and annual radial growth from the photos followed a procedure given by Biging and Wensel (1984) in which a digitizer was used to record the cartesian coordinates of annual ring boundaries from the pith to the outer edge of a section.

There were 7,792 individual measurements of diameter inside bark and height above ground for 1,039 trees (see Table 3). Thus, there were between seven and eight measurements per tree. A seventy-five percent random sample of the 7,792 measurements was selected for development of the taper equation relationship. These data are hereafter referred to as the taper equation data. The remaining twenty-five percent of the measurements were withheld for testing purposes. These data are referred to as the testing data. However, tables 1 and 4 report the coefficients and fit statistics of the taper equation using all the data collected. Random selection of measurements was favored over random selection of trees for two reasons. The primary rationale is that the effect of collinearity associated with sequential use of data points should be minimized by a random selection of measurements.[1/] Additionally, it ensures an equal proportion of measurements in the taper equation data to testing data whereas a random tree selection yields unequal proportions of measurements between data sets. This is because the number of measurements per tree are not constant but vary by trees.

IV. Taper Model

An extensive analysis of the data led to the development of a taper model derived from an S-shaped equation. The model so derived is given as:

$$d = b_1 DBH + b_2 DBH \ln(1 - \lambda(h/H)^{1/3}) \text{ or}$$

$$d = DBH[b_1 + b_2 \ln(1 - \lambda(h/H)^{1/3})] \text{ where}$$

d = predicted diameter inside bark (in.)
b_1, b_2 = regression coefficients
$\lambda = 1 - \exp(-b_1/b_2)$

[1/] However, regression models of tree profile fitted to a 75% sample of ponderosa pine using a random point and random tree selection scheme revealed no significant differences. Coefficients were within 0.1-0.4% of one percent of each other. The estimates of the standard error of the parameters were between 1-2% of each other. Therefore it was concluded that selection scheme had insignificant effect on the resultant estimates of the coefficients and their standard errors.

Table 3. Summary statistics for the stem analysis data base.[1/]

	SPECIES						Total
	PP	DF	RF	WF	SP	IC	
Av. DBH (in)	19.80	16.16	18.26	16.92	20.80	14.92	
S_{DBH} (in)	6.41	6.05	4.10	5.86	6.02	6.10	
min. DBH (in)	5.50	5.70	7.80	5.60	6.20	5.60	
max. DBH (in)	36.10	32.10	27.80	34.30	32.10	36.30	
Av. ht. (ft)	102.78	90.33	85.25	84.03	92.41	60.06	
S_{HT} (ft)	23.87	24.91	12.19	22.71	19.49	19.32	
min. HT (ft)	27.50	38.80	41.50	25.50	36.70	22.40	
max. HT (ft)	153.10	150.60	101.30	133.30	137.70	108.30	
Number of (h,d) pairs	2,014	1,588	312	2,645	692	541	7,792
Number of trees	254	202	43	361	84	95	1,039

[1/] This data was broken into two parts. The first was a 75% sample used in model construction and the remainder was used for model validation.

h = height above ground
H = total height (ft.)
ln(x)= natural logarithm of x

To illustrate the use of this model let us compute diameter inside bark at log lengths of a ponderosa pine tree of DBH = 20 and total height of 100 feet.

Let $b_1 = 1.01959$
$b_2 = .33567$
$\lambda^2 = 1-\exp(-3.03748) = .95205$

then $d = dib = 20[1.01959 + 0.33567 \ln(1-.95205(h/100)^{1/3})]$.

The results of predicting dib at log lengths are presented below.

h	h/H	d
1.0	0.01	18.85
17.5	0.175	15.29
34.0	0.340	13.06
50.5	0.505	10.86
67.0	0.670	8.37
3.5	0.835	5.16
100.0	1.000	0.00

When evaluated at the base (i.e. for h=0) the predicted diameter is b_1DBH giving b_1 = (dib at base)/(DBH). Thus b_1 is the ratio of dib at the base of the tree to DBH. When evaluated at the tip (i.e. h=H) the predicted diameter is zero. Thus this model is constrained to pass through the tip of the tree.

A. Model Fit

The taper model was fit to each of the six species using nonlinear least squares techniques. Coefficients and fit statistics are presented in Table 4. Detailed analysis of bias and standard error are presented in Tables 17 through 22 of part II. In general the models fit quite well and compared favorably with more complex taper models (see section II). The models displayed some slight biases with respect to relative height, but not with respect to total height or DBH classes.

Table 4. Coefficients and fit statistics for the taper equation

	Species					
	PP	DF	RF	WF	SP	IC
b_1	1.01528	1.00689	1.04533	1.04417	1.00923	1.00035
b_2	0.31502	0.33971	0.32970	0.33885	0.32140	0.39311
n	862	896	873	859	896	842
number of trees	254	202	43	361	84	95
R^2	0.966	0.978	0.952	0.952	0.970	0.952
$S_{y.x}$ (in.)	1.438	1.271	1.700	1.534	1.653	1.909

R^2 = Squared multiple correlation coefficient
$S_{y.x}$ = Standard deviation of the residuals about the regression of y on x

Considering all equations jointly the range in bias relations are as follows for the taper equation data (75% sample):

Relative height	range in bias (in.)[1/]	Average across species (in.)
1'-4.5'	-0.06 to 0.62	0.33
4.5'-.1H	-0.78 to -0.45	-0.58
.1H-.2H	-0.50 to 0.24	-0.15
.2H-.4H	-0.24 to 0.61	0.19
.4H-.6H	0.17 to 0.91	0.58
.6H-.8H	-0.02 to 0.37	0.10
.8H-1.H	-0.47 to 0.11	-0.20

These biases although significant are not substantial given the usual level of accuracy in predicting upper stem diameters. Indeed given the fact that one model is used for all height and diameter classes for a given species they performed well.

[1/] Negative values imply overestimates and positive values imply underestimates.

V. Scaling Routines

A. Cubic Volume.

Cubic volume can be determined without the use of scaling routines. This is possible because the cubic volume can be determined using a calculus technique known as generating a solid of revolution. The general form is:

$$V = \int_a^b \pi R^2 dh = \int_a^b \pi \left(\frac{d}{2}\right)^2 dh$$

By substituting

$$d = b_1 DBH + b_2 DBH \ln (1 - (h/H)^{1/3})$$

and using a change of variables we can obtain an expression for cubic volume that allows for directly determining volume obviating the use of scaling routines (see Appendix A).

B. Board Foot Volume

For each DBH and height class in the table the taper equation was used to evaluate the stem profile for a specific size class. That is, by specifying DBH and H we can generate stem profile to predict scaling diameters at log lengths. Board foot scale was performed using a 1.0 foot stump and a log length of 16.5 feet which included a 0.5 foot trim allowance.[2/] After predicting scaling diameters for log lengths, volumes for individual logs were determined using the following equation (Bruce and Schumacher, 1950):

$$V = .79d^2 - 2d - 4 \qquad d = \text{d.i.b. at small end of the log}$$

Top logs less than 16.5 feet were scaled as whole logs and then reduced in volume by a factor of length/16. If the top log was less than 8.5 feet (including trim) the log was assigned a volume of zero.

For example, a 13.7 foot log is considered to have 13/16 of the volume of a whole log whereas a 13.4 foot log contains 12/16 of the whole log volume since it lacks the 6 inch trim requirement of a 13 foot log. The Scribner volume equation was used for scaling diameters of 8 inches and above. For scaling diameters of 6.0 and 7.0 inches the Scribner volume table values of 20.0 and 30.0 board feet, respectively, were used instead of the Scribner volume equation values. This is because the Scribner volume equation is not accurate for trees this small.

VI. Volume Tables

[2/] These specifications were adopted at the October 1980 meeting of the Norther California Forest Yield Cooperative.

Volume tables for each species are presented for the following:

A. Cubic foot volume to a 4 or 6-inch top (Tables 5-10)
B. Board foot volume to a 6 or 8-inch top (Tables 11-16)

Gross cubic-foot volume table to a 1 ft. stump -- 4 inch top

Ponderosa Pine

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	3	4	5	6	7	8	9	10	11	12	13	14
10	6	7	9	11	13	14	16	18	19	21	23	24
12	9	11	14	16	19	21	24	26	29	31	34	36
14	12	16	19	23	26	30	33	37	40	44	47	50
16	16	21	26	30	35	39	44	48	53	57	62	66
18	21	27	33	38	44	50	56	61	67	73	79	85
20	26	33	40	48	55	62	69	76	83	91	98	105
22	32	41	49	58	66	75	84	92	101	110	118	127
24	38	48	59	69	79	90	100	110	121	131	141	152
26	45	57	69	81	93	105	117	130	142	154	166	178
28	52	66	80	94	108	122	136	150	164	179	193	207
30	60	76	92	108	124	141	157	173	189	205	221	237
32	68	87	105	123	142	160	178	197	215	234	252	270
34	77	98	118	139	160	181	201	222	243	264	285	305
36	86	110	133	156	179	203	226	249	273	296	319	342
38	96	122	148	174	200	226	252	278	304	330	356	382
40	107	136	164	193	222	250	279	308	337	365	394	423

Gross cubic-foot volume table to a 1 ft. stump -- 6 inch top

Ponderosa Pine

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	1	2	2	3	3	3	4	4	5	5	6	6
10	4	6	7	8	9	11	12	13	15	16	17	19
12	8	10	12	14	17	19	21	23	26	28	30	32
14	12	15	18	21	24	28	31	34	37	41	44	47
16	16	20	25	29	33	38	42	46	51	55	60	64
18	21	26	32	37	43	49	54	60	66	71	77	83
20	26	33	40	47	54	61	68	75	82	89	96	103
22	31	40	49	57	66	74	83	91	100	108	117	126
24	38	48	58	68	79	89	99	109	120	130	140	150
26	44	56	69	81	93	105	117	129	141	153	165	177
28	52	66	80	94	108	122	136	150	164	178	192	206
30	60	76	92	108	124	140	156	172	188	204	220	236
32	68	86	105	123	141	159	178	196	214	233	251	269
34	77	97	118	139	160	180	201	222	242	263	284	304
36	86	109	133	156	179	202	225	249	272	295	318	342
38	96	122	148	174	200	226	251	277	303	329	355	381
40	107	135	164	193	221	250	279	307	336	365	394	422

Gross cubic-foot volume table to a 1 ft. stump -- 4 inch top

Douglas-Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	3	4	5	6	7	8	9	10	12	13	14	15
10	6	8	9	11	13	15	16	18	20	22	23	25
12	9	12	14	17	19	22	25	27	30	32	35	38
14	13	16	20	24	27	31	34	38	41	45	48	52
16	17	22	26	31	36	40	45	50	55	59	64	69
18	22	28	34	40	46	52	57	63	69	75	81	87
20	27	34	42	49	56	64	71	79	86	93	101	108
22	33	42	51	60	69	77	86	95	104	113	122	131
24	39	50	61	71	82	92	103	114	124	135	146	156
26	46	59	71	84	96	109	121	134	146	159	171	184
28	54	68	83	97	112	126	141	155	170	184	199	213
30	62	78	95	112	128	145	162	178	195	211	228	245
32	70	89	108	127	146	165	184	203	222	241	260	279
34	79	101	122	144	165	186	208	229	251	272	293	315
36	89	113	137	161	185	209	233	257	281	305	329	353
38	99	126	153	179	206	233	260	286	313	340	367	393
40	110	140	169	199	229	258	288	317	347	377	406	436

Gross cubic-foot volume table to a 1 ft. stump -- 6 inch top

Douglas-Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	1	2	2	3	3	4	4	5	5	6	6	7
10	4	6	7	9	10	11	13	14	15	17	18	20
12	8	10	13	15	17	20	22	24	27	29	31	33
14	12	15	19	22	25	29	32	35	39	42	46	49
16	16	21	25	30	34	39	43	48	53	57	62	66
18	21	27	33	39	44	50	56	62	68	74	79	85
20	27	34	41	48	56	63	70	77	85	92	99	106
22	32	41	50	59	68	77	85	94	103	112	121	130
24	39	49	60	71	81	92	102	113	123	134	144	155
26	46	58	71	83	96	108	120	133	145	158	170	182
28	53	68	82	97	111	125	140	154	169	183	198	212
30	61	78	95	111	128	144	161	177	194	211	227	244
32	70	89	108	127	146	164	183	202	221	240	259	278
34	79	101	122	143	165	186	207	229	250	271	293	314
36	89	113	137	161	185	209	232	256	280	304	328	352
38	99	126	153	179	206	233	259	286	313	339	366	393
40	110	140	169	199	228	258	287	317	347	376	406	435

Gross cubic-foot volume table to a 1 ft. stump -- 4 inch top

Red Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	4	5	6	7	8	9	11	12	13	14	15	16
10	7	8	10	12	14	16	18	20	22	24	26	27
12	10	13	16	18	21	24	27	30	32	35	38	41
14	14	18	22	26	29	33	37	41	45	49	53	56
16	18	24	29	34	39	44	49	54	59	64	69	74
18	24	30	36	43	49	56	62	69	75	82	88	94
20	29	37	45	53	61	69	77	85	93	101	109	117
22	36	45	55	65	74	84	94	103	113	123	132	142
24	42	54	65	77	88	100	111	123	135	146	158	169
26	50	63	77	90	104	117	131	145	158	172	185	199
28	58	74	89	105	121	136	152	168	183	199	215	231
30	67	85	103	121	139	157	175	193	211	229	247	265
32	76	97	117	137	158	178	199	219	240	260	281	301
34	86	109	132	155	178	201	225	248	271	294	317	340
36	96	122	148	174	200	226	252	278	304	330	356	382
38	108	136	165	194	223	252	281	310	339	368	396	425
40	119	151	183	215	247	279	311	343	375	407	439	471

Gross cubic-foot volume table to a 1 ft. stump -- 6 inch top

Red Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	2	2	3	4	4	5	6	6	7	8	8	9
10	5	7	8	10	11	13	14	16	18	19	21	22
12	9	12	14	17	19	22	24	27	29	32	34	37
14	13	17	20	24	28	31	35	39	42	46	50	54
16	18	23	28	33	37	42	47	52	57	62	67	72
18	23	29	36	42	48	55	61	67	74	80	86	93
20	29	37	45	52	60	68	76	84	92	100	107	115
22	35	45	54	64	73	83	93	102	112	121	131	140
24	42	54	65	76	88	99	111	122	134	145	156	168
26	50	63	77	90	103	117	130	144	157	171	184	197
28	58	73	89	105	120	136	151	167	183	198	214	230
30	67	84	102	120	138	156	174	192	210	228	246	264
32	76	96	117	137	157	178	198	219	239	260	280	301
34	86	109	132	155	178	201	224	247	270	293	316	340
36	96	122	148	174	200	226	251	277	303	329	355	381
38	107	136	165	194	223	251	280	309	338	367	396	425
40	119	151	183	215	247	279	311	343	375	407	439	471

Gross cubic-foot volume table to a 1 ft. stump -- 4 inch top

White Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	4	5	6	7	8	10	11	12	13	14	15	17
10	7	9	11	13	14	16	18	20	22	24	26	28
12	10	13	16	19	22	24	27	30	33	36	39	42
14	14	18	22	26	30	34	38	42	46	50	54	57
16	19	24	29	34	39	45	50	55	60	65	70	76
18	24	31	37	44	50	57	63	70	76	83	90	96
20	30	38	46	54	62	70	78	87	95	103	111	119
22	36	46	56	66	76	85	95	105	115	125	135	144
24	43	55	67	78	90	102	113	125	137	149	160	172
26	51	65	78	92	106	120	133	147	161	175	188	202
28	59	75	91	107	123	139	155	171	187	203	219	235
30	68	86	105	123	141	160	178	196	215	233	251	270
32	77	98	119	140	161	182	202	223	244	265	286	307
34	87	111	134	158	182	205	229	252	276	299	323	346
36	98	124	151	177	204	230	256	283	309	336	362	389
38	109	139	168	198	227	256	286	315	345	374	404	433
40	121	154	186	219	252	284	317	349	382	415	447	480

Gross cubic-foot volume table to a 1 ft. stump -- 6 inch top

White Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	2	3	3	4	5	5	6	7	7	8	9	9
10	5	7	9	10	12	13	15	17	18	20	21	23
12	9	12	14	17	20	22	25	27	30	33	35	38
14	13	17	21	25	28	32	36	40	43	47	51	55
16	18	23	28	33	38	43	48	53	58	63	68	73
18	23	30	36	43	49	56	62	69	75	81	88	94
20	29	37	45	53	61	69	77	85	93	101	109	117
22	36	46	55	65	75	85	94	104	114	124	133	143
24	43	55	66	78	89	101	113	124	136	148	159	171
26	51	64	78	92	105	119	133	146	160	174	187	201
28	59	75	91	106	122	138	154	170	186	202	218	234
30	68	86	104	122	141	159	177	196	214	232	250	269
32	77	98	119	140	160	181	202	223	244	264	285	306
34	87	111	134	158	181	205	228	252	275	299	322	346
36	98	124	151	177	203	230	256	282	309	335	361	388
38	109	139	168	197	227	256	285	315	344	374	403	432
40	121	154	186	219	251	284	316	349	382	414	447	479

Gross cubic-foot volume table to a 1 ft. stump -- 4 inch top

Sugar Pine

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	3	4	5	6	7	8	9	10	11	12	12	13
10	5	7	9	10	12	14	15	17	18	20	22	23
12	8	11	13	16	18	20	23	25	28	30	33	35
14	12	15	19	22	25	29	32	35	39	42	45	49
16	16	20	25	29	33	38	42	47	51	55	60	64
18	20	26	31	37	43	48	54	59	65	70	76	82
20	25	32	39	46	53	60	67	74	80	87	94	101
22	31	39	47	56	64	72	81	89	98	106	114	123
24	37	47	56	66	76	86	96	106	116	126	136	146
26	43	55	66	78	90	102	113	125	137	149	160	172
28	50	64	77	91	104	118	132	145	159	172	186	200
30	58	73	89	104	120	136	151	167	183	198	214	229
32	66	83	101	119	137	154	172	190	208	226	243	261
34	74	94	114	134	154	174	195	215	235	255	275	295
36	83	106	128	151	173	196	218	241	263	286	308	331
38	93	118	143	168	193	218	243	268	293	319	344	369
40	103	130	158	186	214	242	270	297	325	353	381	409

Gross cubic-foot volume table to a 1 ft. stump -- 6 inch top

Sugar Pine

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	1	2	2	3	3	4	4	5	5	6	6	7
10	4	5	7	8	9	10	12	13	14	15	17	18
12	7	9	12	14	16	18	20	22	24	27	29	31
14	11	14	17	20	23	27	30	33	36	39	42	45
16	15	19	23	28	32	36	40	45	49	53	57	61
18	20	25	30	36	41	47	52	58	63	68	74	79
20	25	31	38	45	52	58	65	72	79	86	92	99
22	30	38	47	55	63	71	80	88	96	105	113	121
24	36	46	56	66	76	86	95	105	115	125	135	145
26	43	54	66	78	89	101	112	124	136	147	159	171
28	50	63	77	90	104	117	131	144	158	171	185	198
30	57	73	88	104	119	135	150	166	182	197	213	228
32	65	83	101	118	136	154	171	189	207	225	242	260
34	74	94	114	134	154	174	194	214	234	254	274	294
36	83	105	128	150	173	195	218	240	263	285	308	330
38	92	117	142	167	193	218	243	268	293	318	343	368
40	103	130	158	186	213	241	269	297	325	352	380	408

Gross cubic-foot volume table to a 1 ft. stump -- 4 inch top

Incense Cedar

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	3	3	4	5	6	7	8	8	9	10	11	12
10	5	6	8	9	11	12	14	15	17	18	19	21
12	8	10	12	14	16	18	21	23	25	27	29	32
14	11	14	17	20	23	26	29	32	35	38	41	44
16	14	18	22	26	30	34	38	42	46	50	54	58
18	18	23	28	34	39	44	49	54	59	64	69	74
20	23	29	35	42	48	54	61	67	73	80	86	92
22	28	35	43	51	58	66	74	81	89	97	104	112
24	33	42	51	61	70	79	88	97	106	115	124	134
26	39	50	60	71	82	93	103	114	125	136	146	157
28	45	58	70	83	95	108	120	132	145	157	170	182
30	52	66	81	95	109	124	138	152	167	181	195	210
32	60	76	92	108	125	141	157	173	190	206	222	239
34	67	86	104	122	141	159	177	196	214	233	251	269
36	76	96	117	137	158	178	199	220	240	261	282	302
38	84	107	130	153	176	199	222	245	268	291	314	337
40	93	119	144	170	195	221	246	271	297	322	348	373

Gross cubic-foot volume table to a 1 ft. stump -- 6 inch top

Incense Cedar

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	1	1	2	2	3	3	3	4	4	5	5	5
10	3	5	6	7	8	9	10	11	12	13	15	16
12	6	8	10	12	14	16	18	20	22	24	25	27
14	10	13	15	18	21	24	27	29	32	35	38	41
16	13	17	21	25	29	32	36	40	44	48	52	55
18	18	23	27	32	37	42	47	52	57	62	67	72
20	22	28	35	41	47	53	59	65	72	78	84	90
22	27	35	42	50	57	65	72	80	87	95	103	110
24	33	42	51	60	69	78	87	96	105	114	123	132
26	39	49	60	70	81	92	102	113	124	134	145	155
28	45	57	70	82	94	107	119	131	144	156	169	181
30	52	66	80	94	109	123	137	151	166	180	194	208
32	59	75	92	108	124	140	156	173	189	205	221	237
34	67	85	104	122	140	158	177	195	213	232	250	268
36	75	96	116	137	157	178	198	219	240	260	281	301
38	84	107	130	153	175	198	221	244	267	290	313	336
40	93	118	144	169	195	220	245	271	296	322	347	372

Gross board-foot volume table to a 1 ft. stump -- 6 inch top

Ponderosa Pine

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	10	12	15	17	20	20	20	20	20	31	33
10	18	20	30	43	48	45	56	71	79	80	80	106
12	30	40	51	66	82	87	102	121	131	134	158	169
14	43	60	79	96	113	141	152	175	198	208	239	256
16	59	89	109	135	160	188	219	247	267	304	324	352
18	78	109	146	185	217	250	287	324	358	398	434	471
20	98	134	187	229	278	325	371	421	464	516	559	602
22	120	169	231	281	346	396	463	517	580	636	698	756
24	144	208	281	344	421	494	563	636	706	779	850	923
26	170	260	343	424	511	592	681	763	852	935	1024	1107
28	207	307	400	499	597	698	798	899	999	1102	1202	1306
30	237	356	456	582	688	813	922	1048	1157	1284	1393	1520
32	266	411	527	670	794	936	1063	1206	1333	1477	1605	1750
34	305	468	603	763	907	1068	1214	1374	1522	1684	1842	2003
36	346	528	683	864	1027	1208	1374	1554	1733	1913	2085	2263
38	390	594	769	969	1155	1355	1545	1745	1946	2146	2342	2538
40	437	662	859	1082	1290	1512	1725	1946	2173	2391	2615	2827

Gross board-foot volume table to a 1 ft. stump -- 8 inch top

Ponderosa Pine

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	15	19	21	22	24	28	30	31
12	21	26	32	36	39	61	68	77	81	84	106	114
14	33	42	66	81	93	101	129	145	155	180	195	208
16	47	78	97	116	149	172	189	224	247	263	298	322
18	63	104	133	174	204	230	274	304	345	378	404	448
20	99	134	171	224	263	316	356	406	449	486	541	582
22	123	169	231	281	329	395	445	512	562	628	680	743
24	148	208	282	344	420	484	558	626	686	768	830	911
26	175	250	336	413	504	581	672	750	841	921	1013	1093
28	204	295	397	488	592	686	792	886	994	1087	1195	1289
30	235	345	460	569	689	799	922	1032	1157	1266	1393	1502
32	267	398	527	656	794	936	1063	1204	1333	1474	1605	1747
34	305	470	603	765	907	1068	1214	1374	1522	1683	1832	1994
36	346	531	683	866	1027	1209	1374	1556	1723	1904	2074	2256
38	390	597	769	973	1155	1358	1545	1748	1936	2140	2330	2535
40	437	665	859	1085	1290	1514	1725	1950	2162	2388	2601	2828

Gross board-foot volume table to a 1 ft. stump -- 6 inch top

Douglas-Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	10	13	16	18	20	20	20	20	31	43	46
10	18	30	40	45	50	46	69	75	80	81	94	110
12	30	41	54	68	84	90	105	125	133	143	164	173
14	44	63	81	101	116	146	159	181	205	215	247	266
16	63	92	114	141	165	196	226	252	276	315	335	373
18	81	108	152	191	225	268	299	336	380	413	449	489
20	102	140	194	233	289	336	385	435	482	534	580	633
22	125	176	241	292	360	411	480	531	601	658	723	781
24	149	216	291	367	436	512	583	660	731	808	879	957
26	176	270	355	440	528	614	704	790	881	968	1058	1147
28	214	318	413	519	618	724	825	932	1033	1142	1238	1352
30	245	370	474	604	713	843	955	1085	1198	1328	1442	1572
32	277	426	547	694	823	970	1101	1248	1381	1528	1671	1809
34	317	485	626	792	940	1105	1257	1423	1576	1742	1907	2071
36	360	548	709	895	1064	1250	1423	1609	1793	1978	2158	2339
38	406	616	798	1005	1197	1402	1599	1805	2015	2218	2424	2619
40	454	686	892	1120	1336	1564	1795	2022	2248	2472	2704	2924

Gross board-foot volume table to a 1 ft. stump -- 8 inch top

Douglas-Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	15	17	21	22	26	28	31	31	32
12	21	28	34	38	56	66	74	80	83	102	113	121
14	34	44	70	85	96	120	137	151	160	189	204	216
16	49	81	103	121	156	179	196	233	256	272	312	333
18	66	109	138	180	212	238	285	316	359	393	419	467
20	104	140	178	233	273	329	370	422	467	504	563	603
22	127	176	241	292	356	410	461	531	583	651	705	771
24	153	216	292	357	435	502	580	649	726	796	859	944
26	181	260	350	429	521	602	697	777	872	954	1049	1132
28	211	307	410	506	614	711	821	917	1030	1125	1237	1335
30	243	358	475	590	713	828	954	1068	1198	1310	1442	1569
32	276	414	547	696	823	969	1101	1248	1381	1526	1661	1808
34	317	488	626	794	940	1106	1257	1423	1576	1743	1896	2062
36	360	551	709	897	1064	1251	1423	1610	1783	1971	2145	2334
38	406	619	798	1008	1197	1405	1599	1808	2004	2213	2410	2620
40	454	690	892	1125	1336	1568	1785	2017	2237	2468	2690	2922

Gross board-foot volume table to a 1 ft. stump -- 6 inch top

Red Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	10	13	16	20	20	20	20	41	45	47	50	50
10	20	30	42	43	47	59	76	82	83	106	109	113
12	30	47	58	72	87	98	125	137	139	168	181	187
14	49	69	86	111	137	159	175	197	225	246	272	289
16	68	98	126	153	185	216	246	278	312	345	367	409
18	88	119	166	206	247	291	328	366	410	451	501	536
20	110	153	212	255	315	365	421	473	528	582	635	690
22	135	192	261	319	391	449	524	581	657	723	790	861
24	162	245	316	400	474	559	644	719	805	882	968	1044
26	200	293	384	478	573	668	764	861	957	1055	1151	1250
28	230	345	447	563	666	787	892	1014	1120	1242	1348	1471
30	260	401	515	656	776	915	1039	1179	1304	1445	1569	1711
32	301	461	595	753	894	1053	1197	1356	1501	1661	1817	1976
34	344	525	679	859	1021	1200	1366	1545	1722	1891	2072	2249
36	390	594	769	970	1155	1355	1545	1746	1947	2146	2343	2539
38	440	666	865	1089	1298	1521	1736	1968	2187	2406	2631	2845
40	492	742	966	1213	1449	1695	1947	2192	2440	2678	2935	3174

Gross board-foot volume table to a 1 ft. stump -- 8 inch top

Red Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	17	21	24	28	31	32	33	34	35	53
12	24	32	38	57	66	77	84	89	112	121	131	137
14	38	49	79	95	107	137	155	167	194	215	228	254
16	54	91	113	133	173	197	233	260	282	319	347	367
18	87	119	151	199	233	279	314	346	396	431	477	516
20	112	153	210	254	299	359	405	464	511	569	619	660
22	139	192	262	319	390	449	504	580	637	711	770	844
24	166	235	319	390	475	548	633	708	793	869	955	1031
26	196	283	379	467	568	656	759	847	950	1040	1144	1234
28	227	334	444	551	666	773	892	999	1119	1226	1347	1454
30	262	389	514	642	776	916	1039	1178	1304	1441	1569	1708
32	301	464	595	755	894	1054	1197	1355	1501	1660	1807	1965
34	344	528	679	861	1021	1202	1366	1547	1712	1892	2061	2242
36	390	598	769	974	1155	1359	1545	1749	1937	2141	2331	2536
38	440	670	865	1091	1298	1524	1736	1963	2176	2403	2617	2845
40	492	746	966	1218	1449	1699	1937	2187	2428	2679	2920	3174

Gross board-foot volume table to a 1 ft. stump -- 6 inch top

White Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	11	13	17	20	20	20	40	42	46	50	50	50
10	20	30	42	45	48	70	77	83	84	104	112	115
12	25	48	59	73	89	100	127	140	142	172	185	190
14	50	70	87	113	139	161	178	201	224	251	279	290
16	69	99	128	156	188	220	250	284	318	352	375	418
18	90	121	169	210	251	297	334	374	418	460	510	547
20	112	156	215	260	321	371	429	482	538	593	647	704
22	137	195	266	325	398	457	533	591	669	727	805	878
24	164	249	321	406	482	569	656	732	820	898	986	1064
26	202	297	390	486	583	680	778	876	975	1074	1173	1272
28	233	350	454	573	677	801	908	1032	1140	1265	1373	1498
30	263	406	523	666	789	931	1057	1199	1327	1471	1598	1743
32	304	468	604	765	909	1071	1217	1379	1528	1691	1849	2002
34	348	532	689	872	1037	1219	1389	1572	1752	1925	2109	2289
36	395	602	781	985	1174	1378	1571	1776	1981	2184	2385	2585
38	445	675	878	1106	1319	1545	1765	1992	2225	2449	2677	2895
40	498	752	980	1232	1473	1724	1980	2230	2482	2724	2987	3230

Gross board-foot volume table to a 1 ft. stump -- 8 inch top

White Fir

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	0	0	0	0	0	0	0	0	0	0	0
10	0	15	19	22	26	30	32	33	34	35	54	58
12	24	33	39	58	69	80	86	91	116	128	135	140
14	39	50	80	97	109	140	158	171	201	219	234	262
16	55	92	116	136	176	201	238	265	288	326	355	375
18	89	121	154	203	237	284	320	354	404	440	487	527
20	113	156	213	259	304	366	413	473	521	580	631	674
22	141	195	266	325	397	457	514	591	649	725	785	861
24	169	239	324	396	483	558	645	721	808	886	971	1051
26	199	287	385	475	578	667	771	863	968	1059	1166	1257
28	230	339	451	560	678	787	908	1017	1140	1248	1372	1481
30	265	395	522	652	789	931	1057	1198	1327	1467	1598	1738
32	304	471	604	767	909	1072	1217	1379	1528	1690	1839	2001
34	348	535	689	874	1037	1222	1389	1572	1742	1926	2097	2282
36	395	606	781	989	1174	1382	1571	1779	1971	2177	2372	2579
38	445	679	878	1109	1319	1549	1765	1995	2213	2445	2664	2896
40	498	756	980	1237	1473	1728	1970	2224	2470	2727	2972	3230

Gross board-foot volume table to a 1 ft. stump -- 6 inch top

Sugar Pine

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	10	12	15	17	20	20	20	20	20	31	43
10	17	20	30	41	46	45	47	68	74	80	81	82
12	30	36	47	55	69	87	91	106	125	134	138	162
14	29	55	77	93	107	129	149	161	181	205	214	244
16	54	75	96	126	155	176	202	227	253	285	314	331
18	71	105	134	167	196	236	272	305	336	381	409	454
20	89	126	173	219	259	301	347	393	435	483	529	574
22	110	153	214	266	323	381	435	496	547	601	660	716
24	132	189	261	319	394	454	531	598	669	738	806	879
26	157	227	312	384	472	545	636	709	801	874	967	1046
28	182	269	366	454	555	654	748	847	943	1042	1138	1239
30	210	325	423	539	654	762	878	987	1105	1215	1332	1444
32	249	373	495	622	749	878	1007	1138	1268	1399	1530	1663
34	281	427	561	710	850	1002	1141	1298	1439	1597	1738	1898
36	314	482	628	803	958	1133	1292	1469	1629	1807	1968	2147
38	348	541	707	903	1077	1273	1453	1649	1831	2029	2212	2411
40	390	605	791	1007	1204	1421	1622	1840	2045	2263	2479	2688

Gross board-foot volume table to a 1 ft. stump -- 8 inch top

Sugar Pine

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	15	17	19	22	24	28	30	31	32
12	19	24	31	35	39	59	67	75	80	84	88	110
14	28	40	47	76	89	99	122	138	151	160	185	200
16	42	71	91	110	125	160	182	197	234	255	270	306
18	57	96	123	147	191	219	257	288	316	358	389	414
20	90	123	159	210	247	281	337	373	424	465	499	555
22	111	153	199	261	309	372	421	481	533	581	645	696
24	136	189	261	319	378	455	515	591	653	727	791	864
26	161	227	312	384	471	545	617	709	782	873	948	1039
28	186	269	367	454	554	644	745	837	938	1032	1118	1226
30	215	315	428	529	647	751	869	975	1096	1202	1322	1430
32	246	363	491	611	744	866	1002	1124	1263	1385	1523	1648
34	278	416	557	698	847	989	1142	1283	1439	1581	1738	1880
36	313	471	629	791	957	1119	1290	1453	1629	1805	1968	2143
38	348	530	707	904	1077	1273	1453	1648	1831	2028	2212	2408
40	390	608	791	1008	1204	1422	1622	1840	2045	2263	2469	2689

Gross board-foot volume table to a 1 ft. stump -- 6 inch top

Incense Cedar

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	0	10	12	13	16	18	20	20	20	20	20
10	15	18	20	30	41	46	50	46	57	71	77	80
12	20	30	40	50	65	78	87	89	103	121	130	134
14	30	47	62	78	93	108	126	147	156	175	198	210
16	36	65	89	109	131	154	177	197	230	247	274	302
18	60	86	107	146	179	206	236	266	300	335	368	392
20	76	115	149	188	218	267	308	348	383	434	468	508
22	95	137	187	240	284	333	374	435	484	536	590	639
24	115	162	229	286	348	412	470	528	593	655	723	782
26	135	196	274	336	417	488	564	639	712	791	863	943
28	159	232	322	398	492	570	665	749	841	928	1019	1108
30	184	272	374	465	573	665	775	869	981	1074	1187	1281
32	209	315	429	538	659	778	892	1012	1129	1249	1367	1488
34	238	370	489	625	750	889	1026	1156	1296	1428	1568	1699
36	267	419	561	708	857	1006	1158	1310	1463	1616	1771	1925
38	308	471	627	795	959	1131	1298	1472	1641	1817	1986	2163
40	341	526	697	889	1065	1263	1444	1644	1827	2028	2213	2415

Gross board-foot volume table to a 1 ft. stump -- 8 inch top

Incense Cedar

DBH (inches)	total height (feet) 40	50	60	70	80	90	100	110	120	130	140	150
8	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	15	19	21	22	24	26	28
12	17	21	26	32	35	38	56	64	69	77	80	84
14	24	34	42	65	76	88	96	118	130	143	153	160
16	36	49	78	96	111	139	159	177	190	223	244	258
18	49	83	107	130	166	193	216	253	282	305	342	371
20	64	108	139	168	216	250	295	331	363	412	448	478
22	80	133	174	229	273	313	372	416	473	520	560	621
24	116	162	214	281	334	401	457	508	580	635	704	762
26	138	196	273	336	402	483	549	630	698	776	849	913
28	162	232	322	398	490	569	649	744	825	921	1002	1097
30	188	272	376	465	571	665	772	869	962	1073	1168	1280
32	216	315	433	538	660	768	892	1002	1126	1239	1362	1477
34	243	360	493	615	753	878	1017	1145	1287	1415	1557	1687
36	274	409	556	697	851	994	1153	1297	1456	1603	1764	1910
38	305	461	624	784	956	1118	1295	1458	1638	1802	1983	2147
40	340	515	696	876	1066	1250	1445	1629	1827	2012	2212	2398

Part II. Taper Equations for Mixed Conifers

I. Model Development

Initial plotting of height versus diameter curves for each species using all the data revealed a significant variance in diameters at given heights. To homogenize the variance, transformations were considered to allow for an improved model fit. Demaerschalk (1971, 1972, 1973) and others (e.g., Max and Burkhart (1976)) have employed relative height and relative diameter as an effective transform.[2/] This study models relative diameter as a function of relative height.[3/]

One objective was to develop a simple model (i.e. one with few parameters) that could accurately describe stem form. Additionally, this equation should be integrable so that cubic foot volumes could be obtained directly, obviating the need for a cubic foot scaling routine. Cubic and board foot volume estimates obtained from the profile equation should be accurate and relatively unbiased.

II. Choice of Model Forms

Much of the work in this field has centered around describing stem curves as polynomials of order two or greater. For example, Kozak et al (1968) used a second order polynomial in relative height; Goulding and Murray (1976) used a fifth order polynomial. The principal weakness of this model type has been the inability to characterize the lower portion of the tree with significant basal swelling (Sterba, 1980). To overcome this problem Bruce et al (1968) used polynomial models with much higher powers to represent the swelling that occurs below DBH.

Alternatively, polynomial regressions with two or more equations can be spliced together to better track stem form. Demaerschalk and Kozak (1977) joined two functions at the inflection point, approximately 20 to 25% of total height from ground level, while maintaining continuity at the join points. Although having a small bias at the base, the equations predicted upper stem diameters well. In a similar fashion Max and Burkhart (1976) and Matney and Sullivan (1980) used segmented polynomials with two and three submodels to predict taper in loblolly pine. This approach produced essentially unbiased models (unbiased over most portions of the stem) and does not require specification of where join points occur.

Demaerschalk (1971, 1972, 1973) and Munro and Demaerschalk (1974) proposed that the mathematical integration of the taper function should be identical to the prediction from existing volume equations. While

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2/ Relative height is the ratio of height above ground to total height and relative diameter is the ratio of diameter inside bark to DBH.

3/ Other transformations of height and diameter were investigated without an increase in predictive power.

this restriction can be imposed upon a taper equation it may degrade the accuracy of diameter predictions. In part this is due to the fact that the taper functions considered as compatible are usually formulated as simple functions of relative height and hence may not track stem form well . For example, Cao et al (1980) reported in their work with loblolly pine that a compatible taper equation does not seem a good choice when describing tree profile since other equations (such as segmented polynomials) were more accurate in predicting upper stem diameters.

Another area of taper research includes functions that inherently have inflection points. Such attempts include use of cosine functions (Alder, 1978), Bitterlich's (1979) use of trigonometric functions, and taper models based on sigmoidal form (Garay (1979), Wensel and Krumland (1983)).

III. Model Derivation

The model chosen to represent tree taper is based on the integral form of the Chapman-Richards function. The applicability of this formulation is presented graphically and the mathematical derivation is outlined.

A. Graphic Derivation

Let H = total height, h = height above ground to some point on the stem and d = diameter at height h. Also, let $y = A(1-\exp(-cx))^m$ where y = 1-h/H and x = d/DBH and A,c and m are parameters to be estimated from the data. This resembles the integral form of the Chapman-Richards function (Garay, 1979; Wensel and Krumland 1983) and can be considered as a model of stem form, as shown in Figure 2, when the axes are properly redefined. By redefining y as h/H one obtains the traditional stem profile (as in Figure 3a). Redefining the base as a - d/DBH forms a mirror image of Figure 3a (as in Figure 3b). The result is that Figure 3b is identical in form to Figure 2. However, the y axis has been redefined to obtain a traditional stem profile in which relative diameter decreases as relative height increases.

Figure 2
Chapman-Richards Function

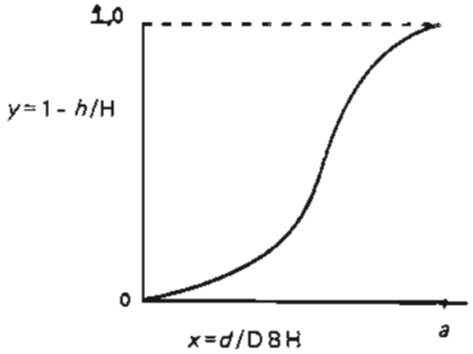


Figure 3a
Stem Profile

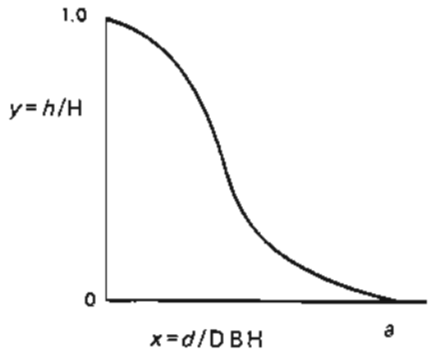
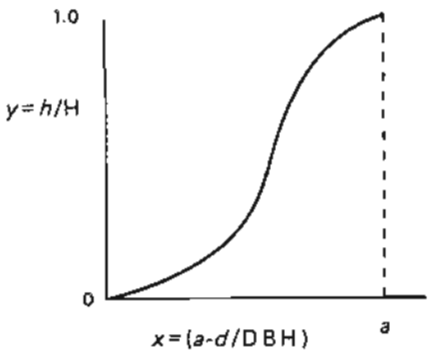


Figure 3b
Stem Profile



B. Mathematical Derivation

Let $y = A(1-\exp(-cx))^m$ where $x = a-d/DBH$, $y = h/H$, $d=dib$, A is the upper asymptote and a = (dib at stump)/DBH. Solving for x yields:

$$x = -\frac{1}{c}\ln[1 - (y/A)^{1/m}] \qquad [1]$$

or equivalently

$$d = DBH[a + \frac{1}{c}\ln(1 - y/A)^{1/m})] \qquad [2]$$

Letting $c' = \frac{1}{c}$ gives

$$d = DBH[a + c'\ \ln(1 - (y/A)^{1/m})] \qquad [3]$$

At the top of the tree, the relative height (h/H) is equal to one

and the diameter (d) should be zero. Substituting these values into equation [3] and solving for A we get:

$$A = (1-\exp(-a/c'))^{-m} \qquad [4]$$

Thus with a constrained model we need to estimate one less parameter.

C. General Model

Substituting expression [4] into equation [3] and letting $b_1 = a$ and $b_2 = c'$ gives:

$$d = DBH[b_1+b_2 \ \ln(1-(h/H)^{1/m}(1-e^{-b_1/b_2}))] \qquad [I]$$

This is the constrained form of the taper equation that is forced to go through the tip of the tree. When evaluated at the base (h=0) the predicted diameter is b_1DBH. Thus b_1 is interpreted as the ratio of dib at the base of the tree to DBH.

Preliminary fitting of model [I] with a 75% sample of Douglas-fir resulted in 1/m = 0.3397 (S.E. = 0.015). When 1/m was restricted to be 1/3 there was a subsequent increase in residual sums of squares of only 16.9 from 1345.25 to 1362.26, a change of only 1%. With a five parameter version of model [I] Wensel and Krumland (1983) reported a 1/m value of .387 for coast redwood. These results coupled with the non-integrability of [I] when m is not an integer led to the fixing of 1/m at 1/3. See Appendix A for integral form of equation [II]. Thus

$$d = DBH[b_1+b_2 \ \ln(1-\lambda(h/H)^{1/3})] \qquad [II]$$

where $\lambda = 1-\exp(-b_1/b_2)$

Analysis and Results

A. Diameter predictions

Model Fit

The sigmoidal taper equation model [II] was fit to each of the six species using nonlinear regression techniques.[4/] Coefficients and fit statistics are presented in Table 1 or Table 4. Detailed analysis of bias, and standard error of the estimate are presented in Tables 17 through 22 for all six species. In general the models fit quite well and compared favorably with more complex models presented in the next section. Model [II] displayed some biases with respect to relative height, but not with respect to total height or DBH.

4/ SAS non-linear regression routines (see Helwig and Council, 1979) were employed using Marquart's compromise for locating the minimum residual sums of squares.

Comparison with Max and Burkhart's Model

Evaluation of equation [II] against Max and Burkhart's (1976) segmented polynomial (termed model [III]) was performed for each species on the same data set used to estimate coefficients for model [II] - the taper equation data. Coefficients and fit statistics for model [III] are presented in Table 23. Cao et al (1980) evaluated several taper equations and found the segmented polynomial to out-perform the others tested in terms of bias, standard error and mean absolute deviation when estimating tree diameters at various points along the stem. In predicting cubic meter volumes to various heights the segmented polynomial model ranked fifth of nine models tested. Max and Burkhart's model (model III) is given as:

$$d^2 = b_1 DBH^2(h/H - 1) + b_2 DBH^2(h^2/H^2 - 1) \qquad [III]$$

$$+ b_3 DBH^2(a_1 - h/H)^2 I_1 + b_4 DBH^2(a_2 - h/H)^2 I_2$$

where $I_i = 1.$ if $h/H \leq a_i$; $i=1,2$

$I_i = 0.$ if $h/H > a_i$

The following criteria were used to evaluate the models in terms of diameters:

1) standard error of the estimate (inches) $\sqrt{\sum(d_i - \hat{d}_i)^2/(n-m-1)}$

2) average bias (inches)[5/] $\sum(d_i - \hat{d}_i)/n$

where $\hat{d}_i$ is predicted diameter at height h_i and d_i is the actual measurement at point "i" on the bole; $m = 2$ = the number of parameters in the model and n is the number of data points in a specified region of relative height, say .1 - .2 of total height.

Results are presented in Tables 17-22. Results presented are similar whether referring to the taper equation data (75% sample) or the testing data (25% sample). Thus unless noted, results apply to both data sets. For the species presented it is evident that the standard errors of the estimate are virtually identical for the two models on every region of the tree from the base to the tip.[6/] This similarity holds across species. However, there are some differences in biases and

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5/ For convenience the subscript i is not included in models II and III, but is implied.

6/ Mean absolute deviation in inches was computed, but conveyed little information above that given by standard error of the estimate. Thus it was not presented in the tables.

Table 17. Statistical evaluation of Models [II] and Model [III].

	Ponderosa pine					
	Standard Error of the Estimate (in.)			Standard Error of the Estimate (in.)		
	Taper Equation Date (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	1.80	1.81	186	1.58	1.61	67
4.5'-.1H	1.46	1.40	186	1.03	1.12	65
.1H-.2H	1.44	1.33	82	1.54	1.63	29
.2H-.4H	1.20	1.18	236	1.13	1.11	67
.4H-.6H	1.35	1.28	228	1.14	1.09	74
.6H-.8H	1.11	1.11	237	1.26	1.24	78
.8H-1.0H	0.84	0.78	380	0.82	0.77	99
TOTAL	1.27	1.22	1535	1.17	1.14	479

	Ponderosa pine					
	Bias (in.)			Bias (in.)		
	Taper Equation Date (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	0.62	0.46	186	0.68	0.55	67
4.5'-.1H	-0.57	0.20	186	-0.39	0.36	65
.1H-.2H	-0.50	0.00	82	-0.49	0.07	29
.2H-.4H	0.17	0.05	236	0.24	0.10	67
.4H-.6H	0.44	-0.03	228	0.41	-0.05	74
.6H-.8H	0.05	-0.07	237	-0.02	-0.10	78
.8H-1.0H	-0.22	-0.02	380	-0.22	-0.04	99
TOTAL	0.03	0.07	1535	0.06	0.11	479

Table 18. Statistical evaluation of Models [II] and Model [III].

	Douglas fir					
	Standard Error of the Estimate (in.)			Standard Error of the Estimate (in.)		
	Taper Equation Date (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	1.14	1.16	149	1.38	1.43	55
4.5'-.1H	1.23	1.15	143	0.81	0.77	49
.1H-.2H	0.93	0.99	38	1.61	1.83	13
.2H-.4H	1.17	1.28	155	1.35	1.52	55
.4H-.6H	1.35	1.59	159	1.40	1.69	43
.6H-.8H	0.99	1.09	176	1.08	1.23	56
.8H-1.0	0.86	0.97	376	0.73	0.91	121
TOTAL	1.07	1.15	1196	1.09	1.18	392

	Douglas fir					
	Bias (in.)			Bias (in.)		
	Taper Equation Date (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	-0.06	-0.10	149	0.28	0.23	55
4.5'-.1H	-0.45	-0.17	143	-0.29	0.00	49
.1H-.2H	-0.01	0.21	38	-0.48	-0.21	13
.2H-.4H	0.51	0.71	155	0.64	0.84	55
.4H-.6H	0.81	1.13	159	0.69	1.03	43
.6H-.8H	0.13	0.42	176	0.26	0.53	56
.8H-1.0H	-0.47	-0.62	376	-0.42	-0.60	121
TOTAL	-0.01	0.08	1196	0.06	0.14	392

Table 19. Statistical evaluation of Models [II] and Model [III].

	Red fir					
	Standard Error of the Estimate (in.)			Standard Error of the Estimate (in.)		
	Taper Equation Data (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	1.10	1.26	34	0.99	1.55	9
4.5'-.1H	1.13	0.79	34	1.51	1.80	7
.1H-.2H	0.02	0.34	2	0.00	0.00	0
.2H-.4H	1.01	1.13	30	1.06	1.66	10
.4H-.6H	1.57	1.63	27	1.46	1.97	10
.6H-.8H	1.30	1.56	31	0.98	1.20	15
.8H-1.0H	1.08	1.52	74	0.87	1.55	29
TOTAL	1.15	1.28	232	1.00	1.19	80

	Red fir					
	Bias (in.)			Bias (in.)		
	Taper Equation Data (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	0.36	-0.52	34	0.49	-0.42	9
4.5'-.1H	-0.78	0.04	34	-1.10	-0.21	7
.1H-.2H	0.09	0.41	2	0.00	0.00	0
.2H-.4H	0.11	0.23	30	0.68	0.80	10
.4H-.6H	0.91	0.76	27	0.63	0.59	10
.6H-.8H	0.00	-0.44	31	0.30	-0.10	15
.8H-1.0H	-0.16	-0.91	74	-0.33	-1.07	29
TOTAL	0.01	-0.30	232	0.06	-0.31	80

Table 20. Statistical evaluation of Models [II] and Model [III].

	White fir					
	Standard Error of the Estimate (in.)			Standard Error of the Estimate (in.)		
	Taper Equation Date (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	1.26	1.31	274	1.21	1.26	86
4.5'-.1H	1.29	1.10	226	1.35	1.10	101
.1H-.2H	1.10	1.29	52	1.60	1.04	24
.2H-.4H	1.18	1.02	274	1.13	1.04	95
.4H-.6H	1.42	1.28	258	1.31	1.21	107
.6H-.8H	1.32	1.34	336	1.50	1.44	100
.8H-1.0H	0.77	0.67	526	0.92	0.84	184
TOTAL	1.17	1.09	1948	1.22	1.12	697

	White fir					
	Bias (in.)			Bias (in.)		
	Taper Equation Date (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	0.08	-0.24	274	0.28	-0.04	86
4.5'-.1H	-0.70	-0.14	226	-0.80	-0.20	101
.1H-.2H	-0.29	-0.35	52	-0.31	-0.38	24
.2H-.4H	0.61	0.14	274	0.56	0.09	95
.4H-.6H	0.67	0.28	258	0.65	0.28	107
.6H-.8H	-0.02	0.40	336	-0.28	0.18	100
.8H-1.0H	-0.31	0.07	526	-0.28	0.15	184
TOTAL	0.01	0.08	1948	-0.03	0.07	697

Table 21. Statistical evaluation of Models [II] and Model [III].

Sugar pine

Height	Standard Error of the Estimate (in.) Taper Equation Date (75% sample) Model [II]	Model [III]	No. of meas.	Standard Error of the Estimate (in.) Testing Data (25% sample) Model [II]	Model [III]	No. of meas.
1.0'-4.5'	1.42	1.44	65	1.37	1.53	20
4.5'-.1H	1.41	1.42	63	1.49	1.71	19
.1H-.2H	1.36	1.55	14	1.37	2.42	8
.2H-.4H	0.99	1.01	80	1.26	1.44	21
.4H-.6H	1.02	0.96	75	1.26	1.18	17
.6H-.8H	1.29	1.53	83	1.03	2.30	30
.8H-1.0H	0.68	1.09	162	1.16	1.64	35
TOTAL	1.08	1.19	542	1.39	1.55	150

Sugar Pine

Height	Bias (in.) Taper Equation Date (75% sample) Model [II]	Model [III]	No. of meas.	Bias (in.) Testing Data (25% sample) Model [II]	Model [III]	No. of meas.
1.0'-4.5'	0.47	0.17	65	0.39	0.18	20
4.5'-.1H	-0.48	0.30	63	-0.39	0.38	19
.1H-.2H	-0.42	-0.31	14	0.03	0.13	8
.2H-.4H	-0.02	-0.26	80	-0.05	-0.39	21
.4H-.6H	0.46	-0.09	75	0.67	0.12	17
.6H-.8H	0.09	-0.61	83	-0.16	-0.90	30
.8H-1.0H	-0.15	-0.78	162	-0.32	-0.97	35
TOTAL	0.02	-0.33	542	-0.03	-0.37	150

Table 22. Statistical evaluation of Models [II] and Model [III].

	Incense Cedar					
	Standard Error of the Estimate (in.)			Standard Error of the Estimate (in.)		
	Taper Equation Date (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	1.14	1.30	86	1.24	1.95	8
4.5'-.1H	1.07	1.07	50	1.63	1.66	19
.1H-.2H	0.69	0.88	21	0.75	2.20	5
.2H-.4H	1.26	1.29	47	0.73	0.79	17
.4H-.6H	1.27	1.33	53	1.39	1.65	13
.6H-.8H	0.95	0.94	75	1.32	1.48	23
.8H-1.0H	0.67	0.78	84	0.57	0.78	40
TOTAL	1.02	1.06	416	1.04	1.04	125

	Incense Cedar					
	Bias (in.)			Bias (in.)		
	Taper Equation Date (75% sample)			Testing Data (25% sample)		
Height	Model [II]	Model [III]	No. of meas.	Model [II]	Model [III]	No. of meas.
1.0'-4.5'	0.54	0.39	86	0.68	0.37	8
4.5'-.1H	-0.49	0.37	50	-0.55	0.32	19
.1H-.2H	0.24	0.49	21	-0.01	0.33	5
.2H-.4H	-0.24	-0.20	47	0.00	0.04	17
.4H-.6H	0.17	0.07	53	0.74	0.68	13
.6H-.8H	0.07	0.13	75	0.20	-0.01	23
.8H-1.0H	0.11	-0.27	84	0.01	-0.37	40
TOTAL	0.15	0.10	416	0.09	0.04	125

Table 22. Coefficients and fit statistics for Max and Burkhart's model (Model III)

	SPECIES					
	PP	DF	RF	WF	SP	IC
b_1	-2.9528	-2.9543	-2.9735	-2.6789	-2.9181	-2.9163
b_2	1.4175	1.3691	1.2973	1.2778	1.3756	1.4190
b_3	-1.6568	-0.9903	-1.1479	-1.7449	-1.1496	-0.9469
b_4	53.2524	29.9328	108.2100	75.0475	76.3697	113.4100
a_1	0.7299	0.9064	0.8864	0.5850	0.8309	0.9040
a_2	0.0909	0.0822	0.0695	0.0734	0.0691	0.0669
n	1,535	1,194	232	1,948	542	410
R^2	0.9717	0.9730	0.9820	0.9769	0.9891	0.9774

R^2 = Squared multiple correlation coefficient computed for d^2 based on analogy to the linear model.

direction of bias. To help summarize the relationship of bias to relative height, the average model behavior across species is presented in Table 24a and Table 24b. There are regions in which the models (on the average) displayed relatively little bias. These were 10-20%, 20-40% and 60-80% of total height. However, the directions of the average biases were usually different for model [II] versus model [III] in these regions.

Between 1'-4.5', 40-60% of tree height and 80-100% of tree height, model [II] and model [III] displayed the same direction of bias. In these regions model [III] had greater variation (approximately 50% higher) in the range of biases across species, but model [II] usually had higher average bias values with the exception being in the highest segment of the tree (80-100% of tree height) where it was lower than model [III].

For the region 4.5'-.1H model [II] showed differences in direction of bias and in the degree of bias from model [III]. In this segment, model [III] was less biased on the average, but showed a range in bias approximately 50% greater than model [II] on the taper equation data set whereas on the testing data the range in bias was 25% less than for model [II]. Model [II] tended to overpredict to a much larger extent than did model [III] for this segment.

In general, both models tended to underpredict at the base (1'-4.5') and at midrange portions of the stem (20-80% of total height). Overpredictions tended to occur at the tip (80-100% of total tree height). Near breast height (4.5'-10% of total height) model [II] overpredicted whereas model [III] underpredicted to a lesser degree. Neither model displayed any bias with respect to DBH or total height.

B. Comparisons of scaled versus predicted volumes

Because taper equations may be used for volume estimation as well as for predicting upper stem diameters, a comparison of volumes calculated using the profile from models [II] and [III] was compared with actual scale[7/] of all trees in the data set.[8/] Using profile equations [II] and [III] cubic volumes were determined by integration (see Appendix A for integration of model [II]). Board foot values were calculated from diameters at log-length intervals by applying the Scribner volume equation (see footnote 7).

7/ Board foot scaling was performed using a 1.0 foot stump and a log length of 16.5 feet which included a 0.5 foot trim allowance. Volumes for individual logs were determined using the following equation (Bruce and Schumacher, 1950): $V = .79d^2 - 2d - 4$ where d = diameter inside bark at the small end of the log. Cubic foot scaling was accomplished using Smalian's formula for geometric solids.

8/ The entire data set was comprised of the taper equation data (75% sample) and the testing data (25% sample) described under Data Sources.

Table 24a. Comparison of bias for Model [II] and Model [III]. Values are presented for the taper equation data (75% sample). Values presented are computed across species.

	Taper Equation Data (75% sample)							
	Model [II]				Model [III]			
Height	Mean	Min	Max	Range	Mean	Min	Max	Range
1.0'-4.5'	0.33	-0.06	0.62	0.68	0.03	-0.52	0.46	0.98
4.5'-.10	-0.58	-0.78	-0.45	0.33	0.10	-0.17	0.37	0.54
.10-.20	-0.15	-0.50	0.24	0.74	0.07	-0.35	0.49	0.84
.20-.40	0.19	-0.24	0.61	0.85	0.11	-0.20	0.71	0.97
.40-.60	0.50	0.17	0.91	0.74	0.35	-0.09	1.13	1.22
.60-.80	0.10	-0.02	0.37	0.39	-0.03	-0.61	0.42	1.03
.80-1.00	-0.20	-0.47	0.11	0.58	-0.42	-0.91	0.07	0.98

Table 24b. Comparison of bias for Model [II] and Model [III]. Values are presented for the testing data (25% sample). Values presented are computed across species.

	Testing Data (25% sample)							
	Model [II]				Model [III]			
Height	Mean	Min	Max	Range	Mean	Min	Max	Range
1.0'-4.5'	0.47	0.28	0.68	0.40	0.14	-0.42	0.55	0.97
4.5'-.10	-0.59	-1.10	-0.29	0.81	0.10	-0.26	0.38	0.64
.10-.20	-0.21	-0.49	0.03	0.52	-0.01	-0.38	0.33	0.71
.20-.40	0.34	-0.05	0.68	0.73	0.25	-0.39	0.84	1.23
.40-.60	0.64	0.41	0.74	0.33	0.42	-0.05	1.03	1.08
.60-.80	0.05	-0.28	0.30	0.58	-0.07	-0.90	0.53	1.43
.80-1.00	-0.26	-0.42	0.01	0.43	-0.48	-1.07	0.15	1.22

Percent overprediction by 4-inch diameter classes was computed and is presented for cubic volume in Table 25 and board foot volume in Table 26. When judged overall DBH classes, model [II] and model [III] performed similarly. For cubic foot volume to a 4-inch top, model [II] underpredicted by 0.3, 4.4, 2.8, 4.6, 1.1 and 3.9 for PP, DF, RF, WF, SP and IC, respectively. Model [III] underpredicted by 1.6, 9.0, 3.4, 3.3, -2.4, 4.9 percent, respectively, for PP, DF, RF, WF, SP, and IC. Within a diameter class there was variation, with models [II] and [III] sometimes underpredicting and sometimes overpredicting. However, the two models almost always predicted in the same direction and were within several percentage points of each other. When the two models were not close together, Model [III] was usually more biased than model [II].

Judged overall, comparisons of board foot predictions to a 6-inch top with scaled values showed model [II] overpredicted 1.6, -6.7, -5.1, -7.3, -0.8 and 0.1 percent for PP, DF, RF, WF, SP and IC, respectively. Model [III] overpredicted 2.3, -11.8, -4.4, -2.4, 6.5 and 1.5 percent for PP, DF, RF, WF, SP, and IC, respectively. As for cubic volumes, within each diameter class, the direction of the differences between scaled and predicted values were similar for both models. For any given diameter class, both models were usually within 5 percentage points of one another. When this difference exceeded 5 percentage points model [III] was usually more biased than model [II].

C. Comparison of taper-based volume to whole tree volume equations

An alternative to taper-based volume estimates are whole tree volume equations. These models can not be used to predict volume by section, but might provide better estimates of whole tree volume. The form of these models for these comparisons (see Wensel, 1977) was:

$$\ln(V) = c_0 + c_1 \ln(DBH) + c_2 \ln(H) \qquad [IV]$$

where ln = natural logarithm and H = total height in feet.

Model [IV] was fit to each species using the scaled volume data previously described. Percent overpredictions for each species by diameter class were computed and are presented in Tables 25 and 26 for cubic and board foot predictions. As judged overall, the cubic foot equations for model [IV] were usually within 0-2% of actual whereas board foot volumes were within 2-4% of actual. In general taper-based predictions performed nearly as well as whole tree equations. As judged overall, Model [II] cubic foot predictions were within 0-5% of actual and board feet predictions were within 0-7% of actual. Model [III] cubic foot and board foot predictions were within 2-10% of actual. Within individual diameter classes, model [IV] performed similarily to model [II] and model [III]. However, in the smaller diameter classes (<20 inches) model [IV] tended to have smaller average bias than the taper-based models. This trend tended to hold more for cubic volume predictions than for board foot predictions. However, considering the small average volumes associated with diameters less than 20 inches it is unlikely that there is a strong advantage to using model [IV] over model [II] or model [III]. These relationships can be easily seen in Figures 4-15 and 16-27. These figures display actual scaled volumes versus predicted

Table 25. Comparison of predicted and scaled cubic volumes to a 4" merchantable top. Average percent over-prediction by DBH class.

Percent Overpredictions[1,2]
DBH Class

Species	Model	8	12	16	20	24	28	32	36	overall
PP	II	-10.3	-2.4	-0.8	-1.3	1.3	4.0	9.2	-0.6	-0.3
	III	-10.4	-3.6	-2.3	-2.7	-0.1	2.7	7.8	-1.9	-1.6
	IV	-0.6	3.0	1.1	-0.8	0.7	2.7	6.8	-0.9	1.2
	n	10	35	62	51	42	28	8	3	239
	av. vol	11.2	20.9	40.8	74.8	115.8	169.3	237.4	316.6	82.1
DF	II	-16.3	-8.2	-5.0	-3.3	3.4	1.8	17.0	-	-4.4
	III	-19.5	-12.5	-9.7	-8.1	-1.6	-3.1	11.4	-	-9.0
	IV	0.9	3.0	0.5	-0.8	3.3	-0.7	11.5	-	1.3
	n	7	44	53	35	16	8	3	-	166
	av. vol.	11.7	20.9	45.2	78.8	116.9	169.6	196.6	-	60.1
RF	II	-	-16.0	0.5	-2.2	6.5	0.1	-	-	-2.8
	III	-	-16.4	-0.1	-2.8	5.8	-0.5	-	-	-3.4
	IV	-	-6.7	5.4	0.3	5.9	-3.6	-	-	1.0
	n	-	7	12	18	4	1	-	-	42
	av. vol.	-	24.5	43.7	72.8	97.3	148.7	-	-	60.6
WF	II	-7.6	-10.0	-6.2	-3.4	3.7	7.6	1.0	6.7	-4.6
	III	-5.8	-8.7	-5.0	-2.0	5.2	9.2	2.6	8.4	-3.3
	IV	7.5	0.3	-0.5	-0.4	3.8	6.3	-2.3	2.1	0.7
	n	8	78	87	83	32	14	3	1	306
	av. vol.	9.2	21.8	45.9	80.2	114.1	168.7	269.6	282.2	63.8
SP	II	-	-8.4	-4.3	3.7	-2.8	2.9	11.5	-	-1.1
	III	-	-4.7	-0.9	7.2	0.5	6.3	15.3	-	2.4
	IV	-	-0.1	-0.1	5.2	-3.6	0.4	7.9	-	0.6
	n	-	12	16	18	19	12	3	-	80
	av. vol.	-	17.9	35.4	59.7	106.5	144.3	177.6	-	76.8
IC	II	-11.6	-9.9	-3.4	0.1	4.8	17.1	-	13.2	-3.9
	III	-9.8	-10.4	-4.8	-1.4	3.2	15.4	-	11.7	-4.9
	IV	10.0	1.0	-0.1	-0.6	0.8	9.7	-	2.0	0.9
	n	2	28	22	15	4	3	-	1	75
	av. vol.	5.5	12.2	26.2	44.5	64.0	92.4	-	194.6	31.0

1\ Percent overprediction = $\frac{\text{Predicted volume - scaled volume}}{\text{scaled volume}} \times 100$

2\ Only trees greater than 8 inches in DBH were scaled. Thus there were fewer trees for analysis than listed in Table 1.

Table 26. Comparison of predicted and scaled board foot volumes to a 6" merchantable top. Average percent over-prediction by DBH class.

Percent Overpredictions[1,2]

DBH Class

Species	Model	8	12	16	20	24	28	32	36	overall
PP	II	-1.1	7.0	0.2	-1.3	0.1	4.2	9.9	0.3	1.6
	III	8.4	4.0	2.0	-0.4	0.6	4.3	9.7	-0.1	2.3
	IV	10.7	10.9	-0.5	-2.9	-0.1	6.5	15.7	8.9	2.7
	n	10	35	62	51	42	28	8	3	239
	av. vol.	40.1	85.0	197.5	407.4	684.6	1043.0	1523.4	2054.5	471.6
DF	II	-27.4	-10.9	-7.4	-4.8	3.9	0.5	19.6	-	-6.7
	III	-7.7	-18.2	-13.3	-10.5	-1.8	-5.2	12.9	-	-11.8
	IV	5.4	5.9	-0.7	-1.1	6.3	1.9	19.9	-	2.4
	n	7	44	53	35	16	8	3	-	166
	av. vol.	45.0	89.7	232.0	439.4	683.9	1055.8	1223.8	-	331.3
RF	II	-	-18.8	-2.0	-5.1	8.1	0.5	-	-	-5.1
	III	-	-16.5	-1.5	-4.6	8.3	0.1	-	-	-4.4
	IV	-	-4.7	4.1	0.0	10.7	1.2	-	-	1.4
	n	-	7	12	18	4	1	-	-	42
	av. vol.	-	109.3	218.5	400.7	545.3	896.6	-	-	[illegible]
WF	II	8.1	-14.8	-9.4	-6.4	3.1	7.6	1.2	9.7	-7.3
	III	15.8	-8.5	-4.9	-2.1	7.1	11.1	4.3	13.1	-2.4
	IV	31.4	0.3	-1.5	-1.1	6.5	13.1	7.3	15.5	1.6
	n	8	78	87	83	32	14	3	1	306
	av. vol.	29.5	95.4	233.5	450.4	664.9	1032.7	1715.9	1763.7	353.0
SP	II	-	-10.2	-2.0	4.6	-3.9	3.6	13.6	-	-0.8
	III	-	1.5	4.2	12.0	2.0	10.0	20.3	-	6.5
	IV	-	1.7	0.9	4.6	-4.9	2.2	14.5	-	1.2
	n	-	12	16	18	19	12	3	-	80
	av. vol.	-	68.8	160.9	303.8	600.0	842.2	1049.9	-	419.0
IC	II	31.0	-6.0	-0.7	1.2	7.5	21.0	-	21.5	0.1
	III	-	-4.3	3.6	1.8	7.6	18.5	-	20.7	1.5
	IV	50.3	4.9	-0.4	-1.1	2.8	17.7	-	23.1	4.0
	n	2	28	22	15	4	3	-	1	75
	av. vol.	10.0	40.1	107.2	205.3	304.4	81.8	-	1076.1	137.6

1\ Percent overprediction = $\frac{\text{Predicted volume - scaled volume}}{\text{scaled volume}}$ X 100

2\ Only trees greater than 8 inches in DBH were scaled. Thus there were fewer trees for analysis than listed in Table 1.

volumes from models [II] and model [IV] for cubic volumes to a 4-inch merchantable top (Figure 4-15) and for board foot volumes to a 6-inch merchantable top (Figures 16-27). The graphs show that the taper-based volume estimates perform well and can be used in lieu of whole tree equations with little loss of accuracy. For the smaller diameter classes, however, the whole tree equations tend to provide better estimates - especially for cubic volume.

Conclusions

In this study, a model derived from a sigmoid equation (model II) was tested on six conifer species of northern California and compared with the segmented polynomial model. As judged by standard error of the estimate, for all species considered this sigmoid model compared closely with the segmented polynomial that was judged superior by Cao et al (1980) in predicting upper stem diameter. There were some differences in direction of bias. The degree of bias was similar for both models, although the levels varied by relative height and species.

In light of the similarity in performance of these two models, the 2-parameter sigmoidal model (model [II]) is preferable because it is more parsimonious than the 6 parameter segmented polynomial model (model [III]). Both model [II] and model [III] can be integrated (see Appendix A.) to directly yield cubic foot volume estimates to any height.

Volumes predicted with the taper based system (models [II] and [III]) were compared with scaled volumes of all trees in the data base. Judged overall, predicted cubic foot volumes were generally within 0 to 5 percent of being unbiased, but for some species over or underprediction of up to 7 percent occurred. The board foot predictions compared to scaled were generally within 2-7% of scaled. For either cubic or board foot volumes, model [II] and model [III] displayed the same pattern and direction of over and underprediction. The two models were usually within 1-5 percentage points of each other regardless of diameter class. When they were not within 1-5 percentage points of each other model [II] was twice as likely as model [III] to be closer to being unbiased. That is, when there was a discrepancy in bias, model [III] was usually more biased than model [II]. Thus, both models performed similarly over the range of data collected. A comparison of scaled to predicted values from whole tree volume equations, with minor exception, yielded similar results in terms of percent overpredictions as judged overall and within diameter classes. Some exceptions were noted for the smaller diameter classes. Thus, it appears that for this data set the taper-based volume estimates can be used in lieu of whole tree volume equations with little or no loss of accuracy.

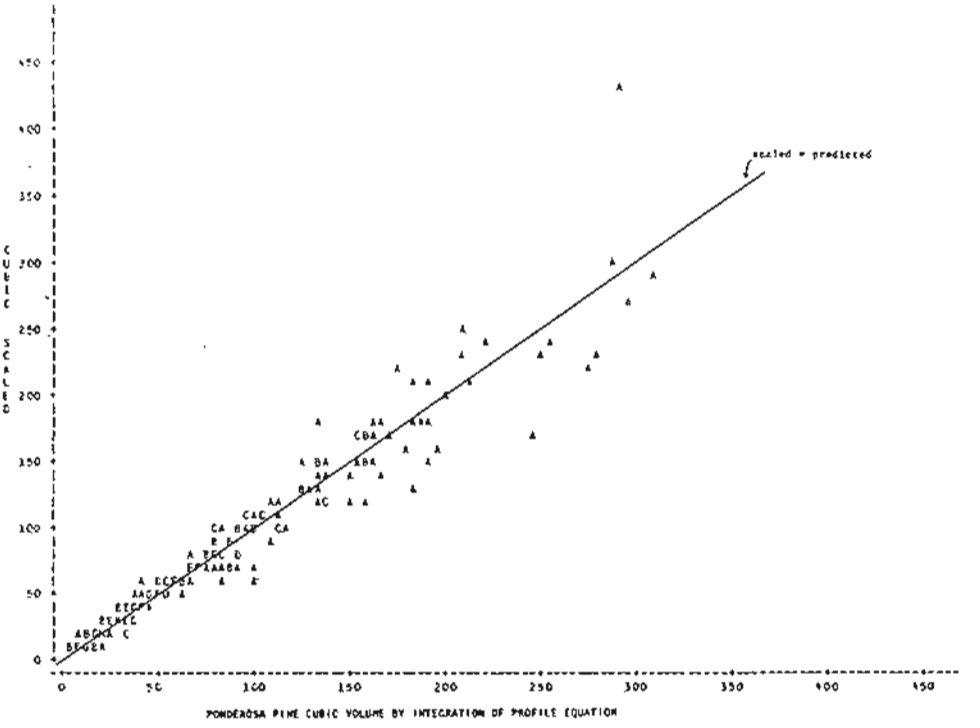

FIGURE 4. PREDICTED CUBIC VOLUME VERSUS SCALED FOR PONDEROSA PINE.

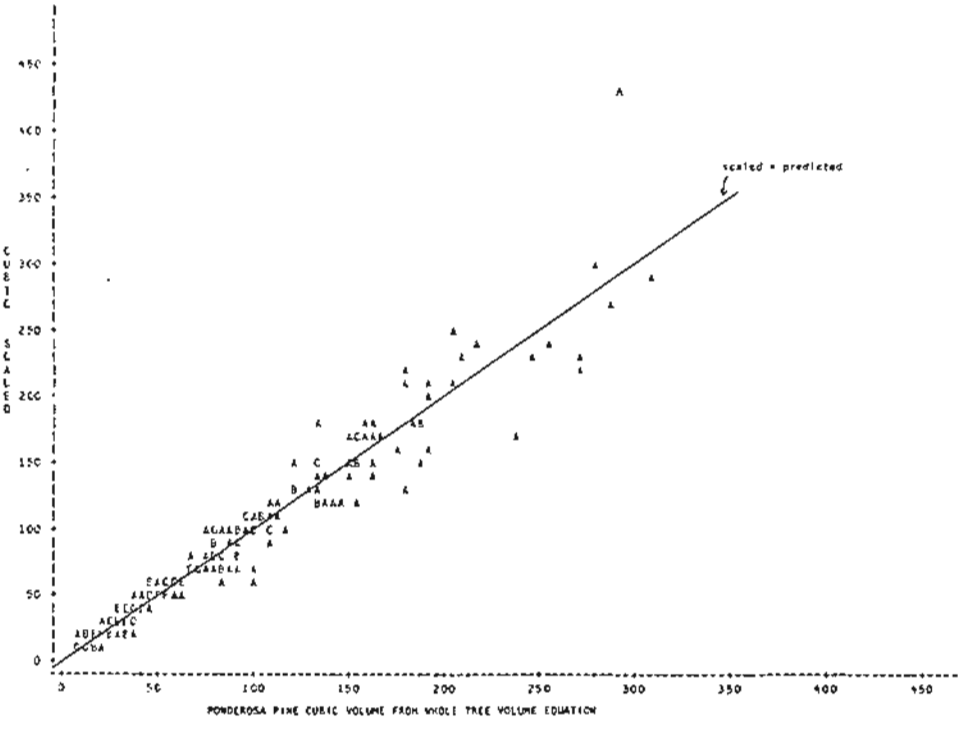

FIGURE 5. PREDICTED CUBIC VOLUME VERSUS SCALED FOR PONDEROSA PINE.

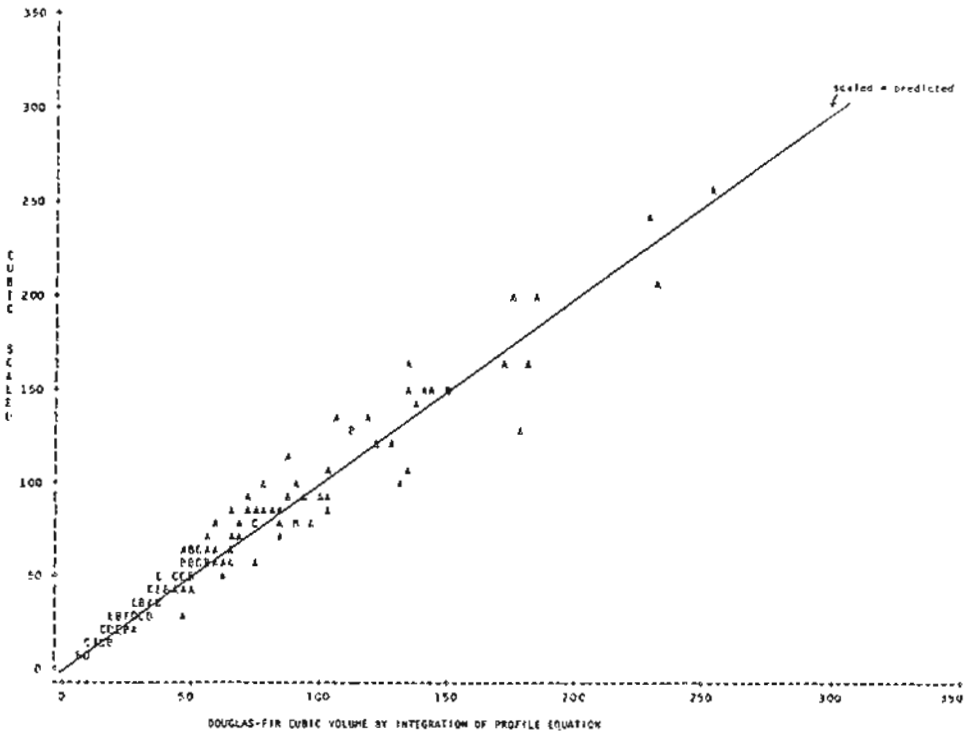

FIGURE 6. PREDICTED CUBIC VOLUME VERSUS SCALED FOR DOUGLAS-FIR.

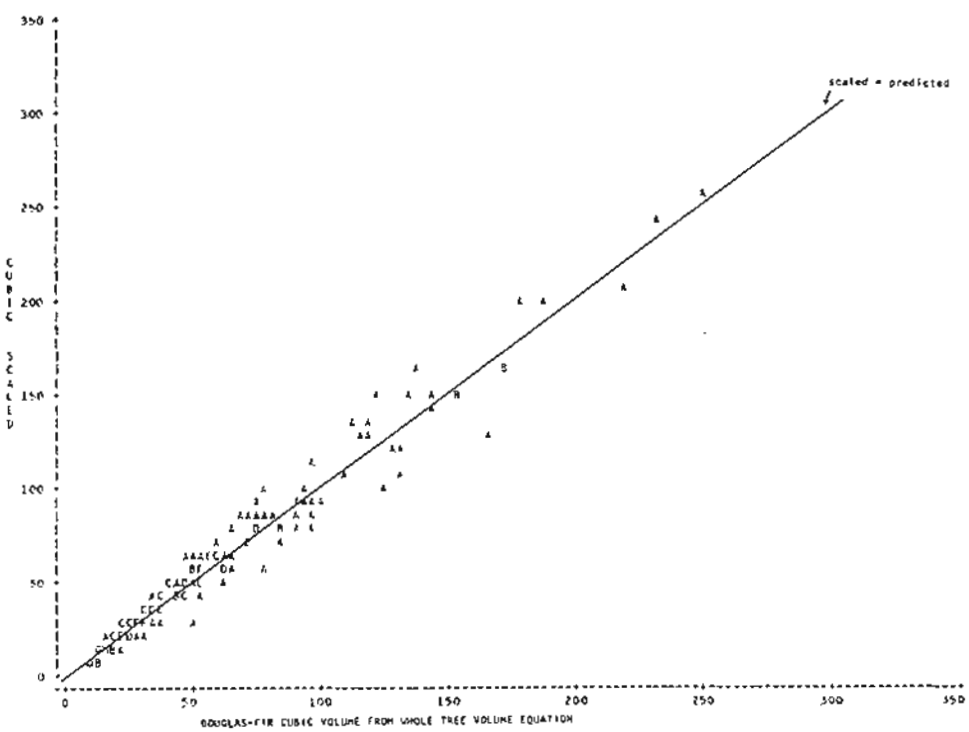

FIGURE 7. PREDICTED CUBIC VOLUME VERSUS SCALED FOR DOUGLAS-FIR.

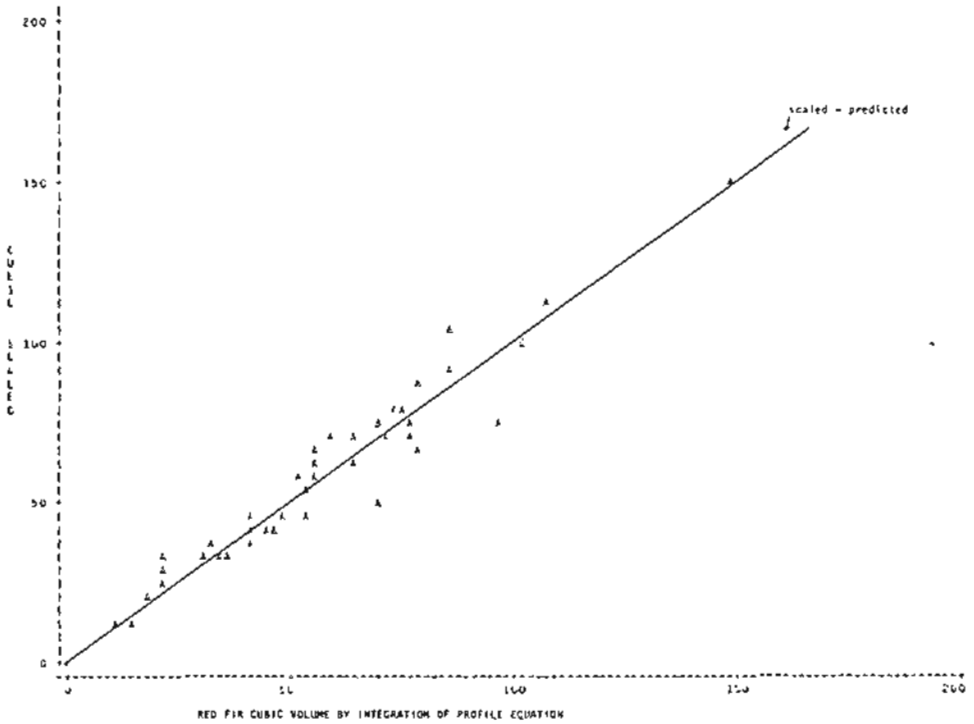

FIGURE 8. PREDICTED CUBIC VOLUME VERSUS SCALED FOR RED FIR.

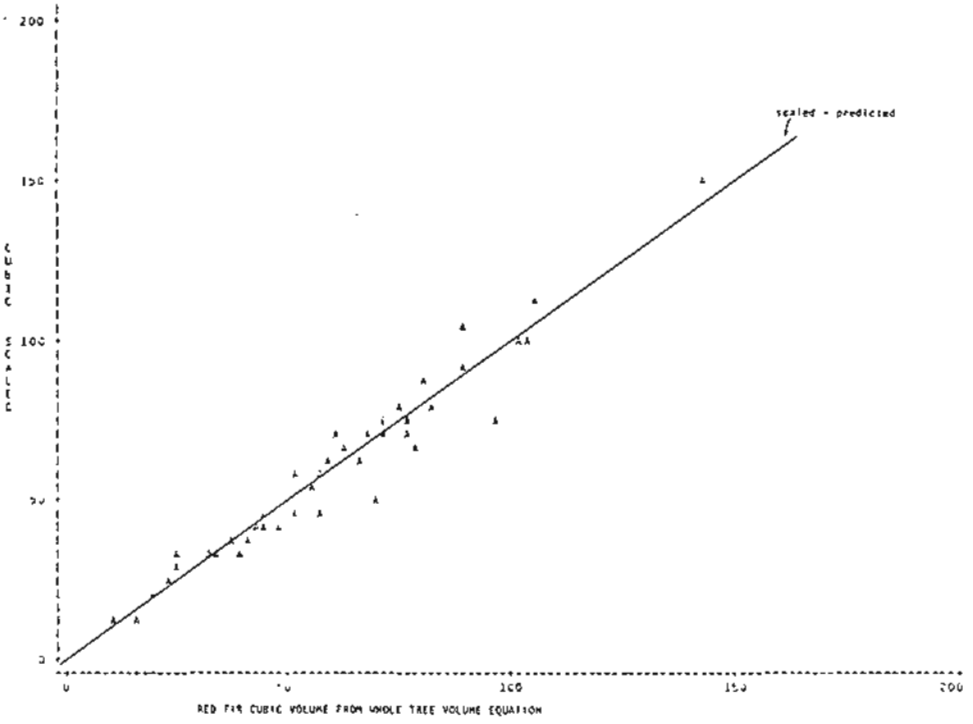

FIGURE 9 PREDICTED CUBIC VOLUME VERSUS SCALED FOR RED FIR.

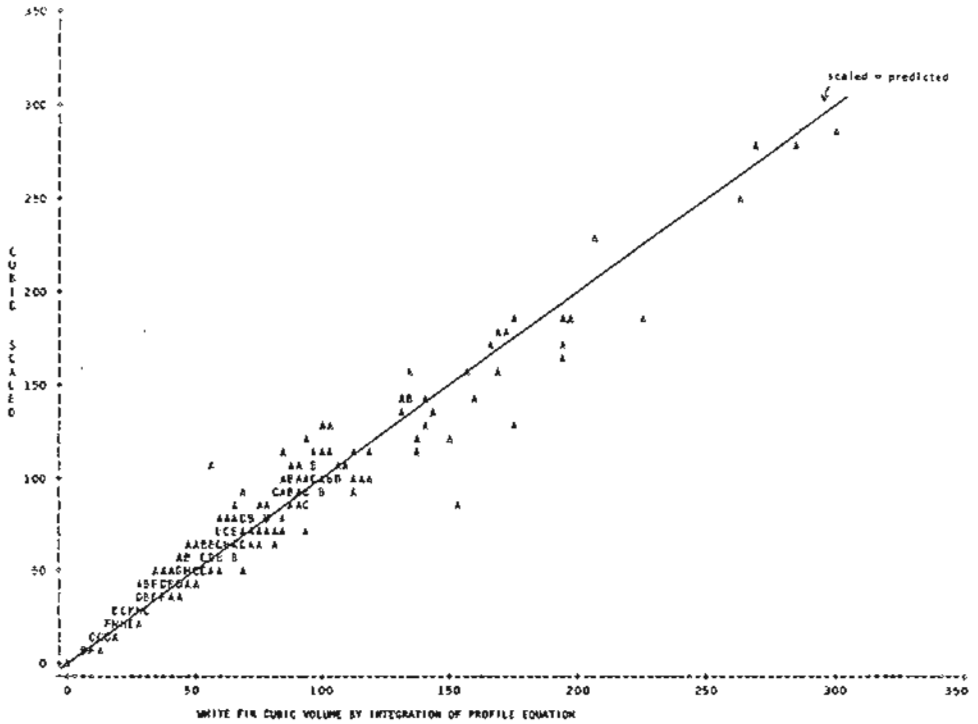

FIGURE 10. PREDICTED CUBIC VOLUME VERSUS SCALED FOR WHITE FIR.

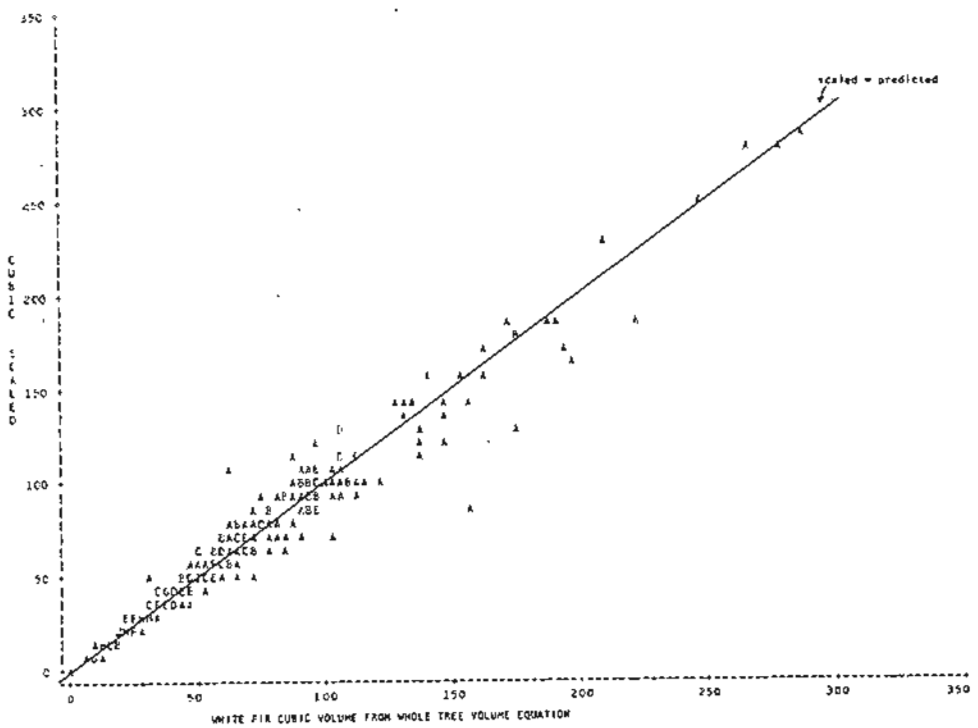

FIGURE 11. PREDICTED CUBIC VOLUME VERSUS SCALED FOR WHITE FIR.

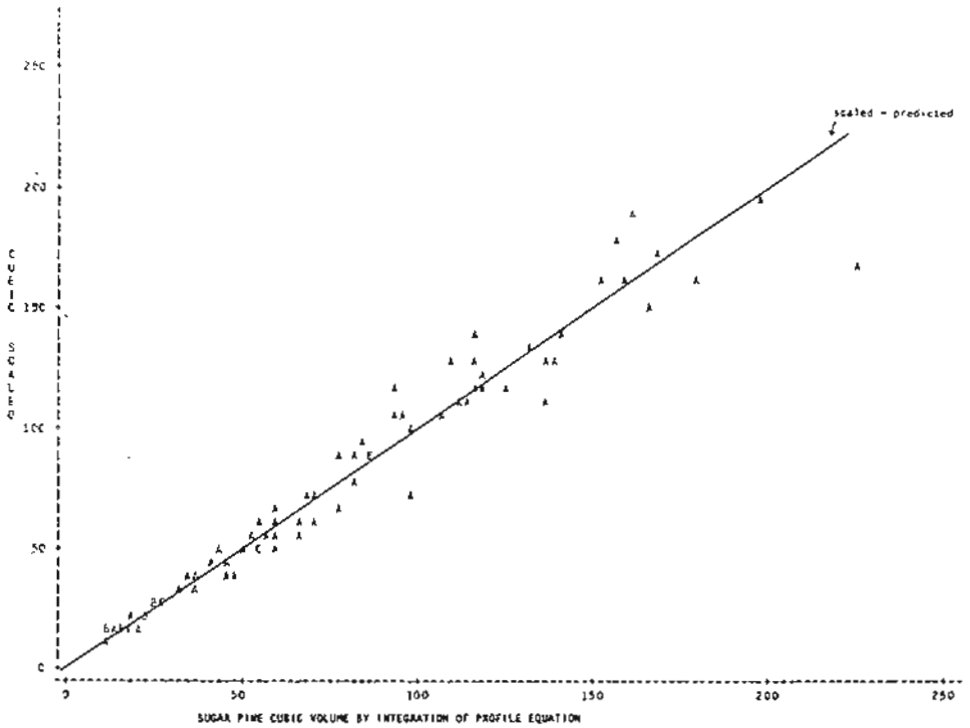

FIGURE 12. PREDICTED CUBIC VOLUME VERSUS SCALED FOR SUGAR PINE.

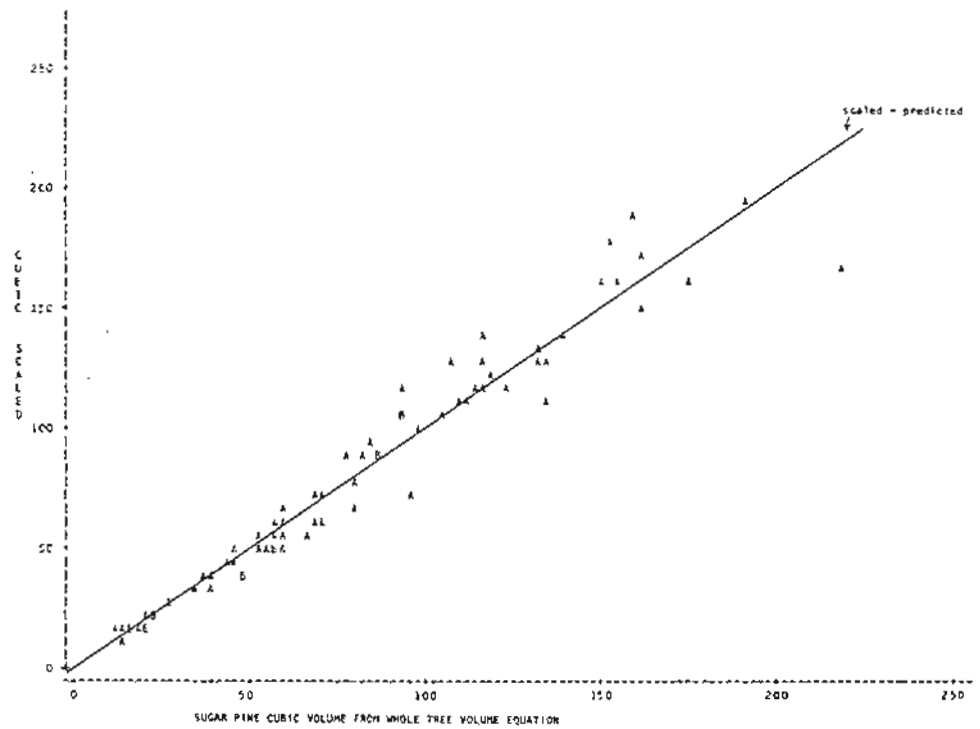

FIGURE 13. PREDICTED CUBIC VOLUME VERSUS SCALED FOR SUGAR PINE.

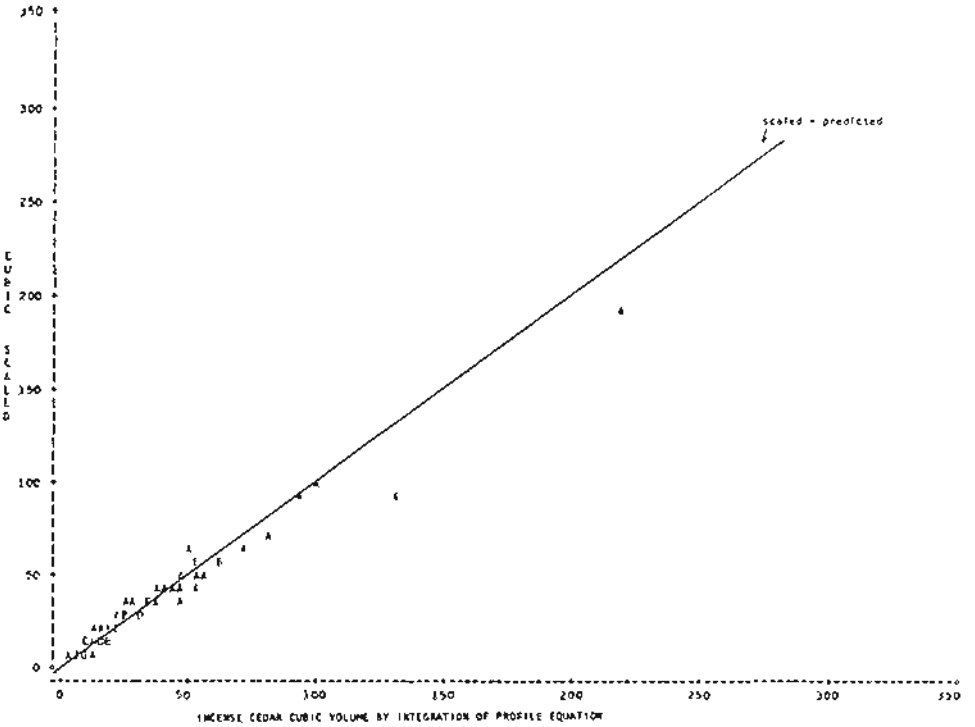

FIGURE 14. PREDICTED CUBIC VOLUME VERSUS SCALED FOR INCENSE CEDAR.

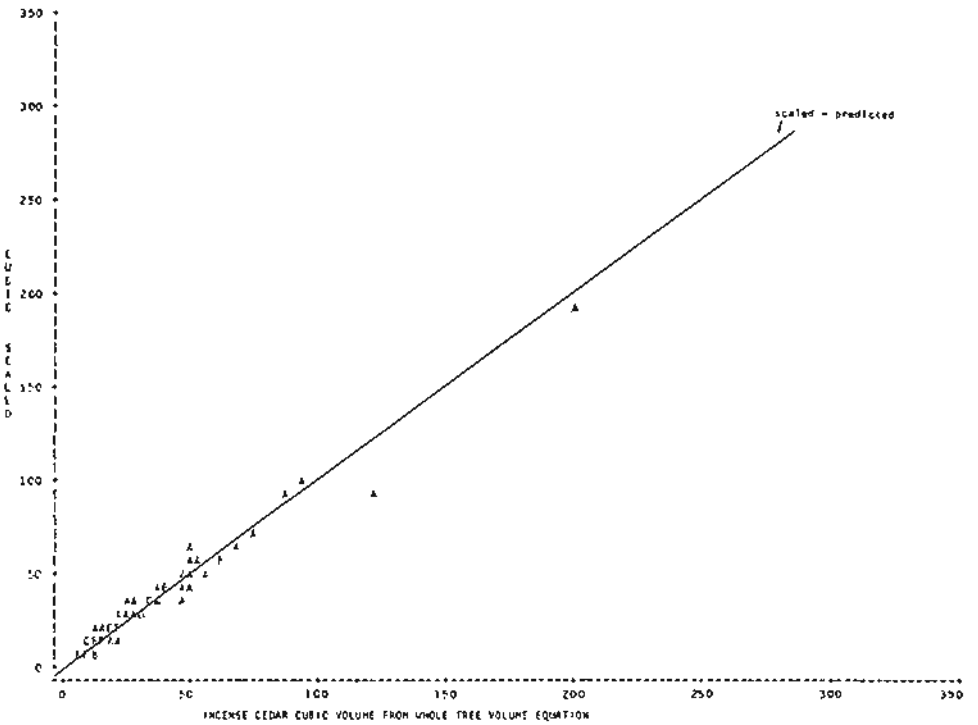

FIGURE 15. PREDICTED CUBIC VOLUME VERSUS SCALED FOR INCENSE CEDAR.

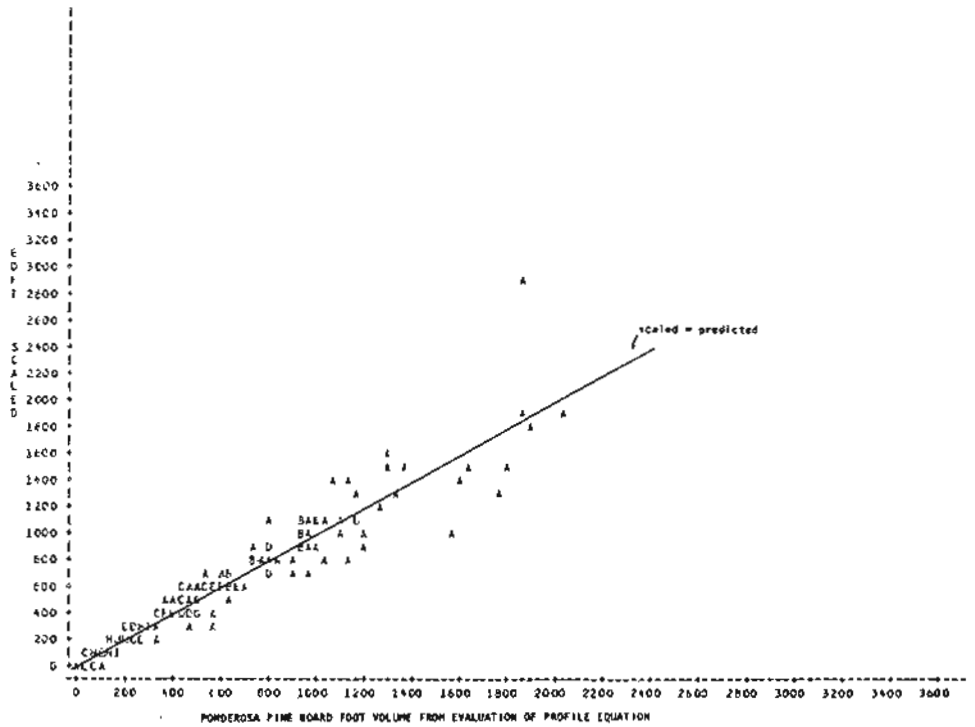

FIGURE 16. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR PONDEROSA PINE.

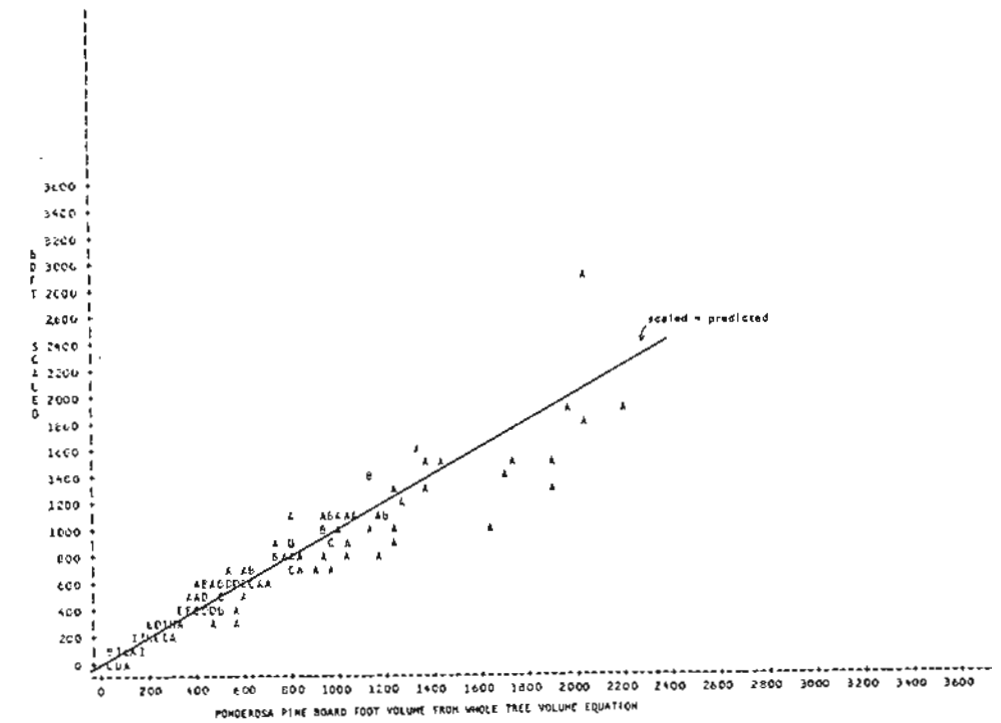

FIGURE 17. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR PONDEROSA PINE

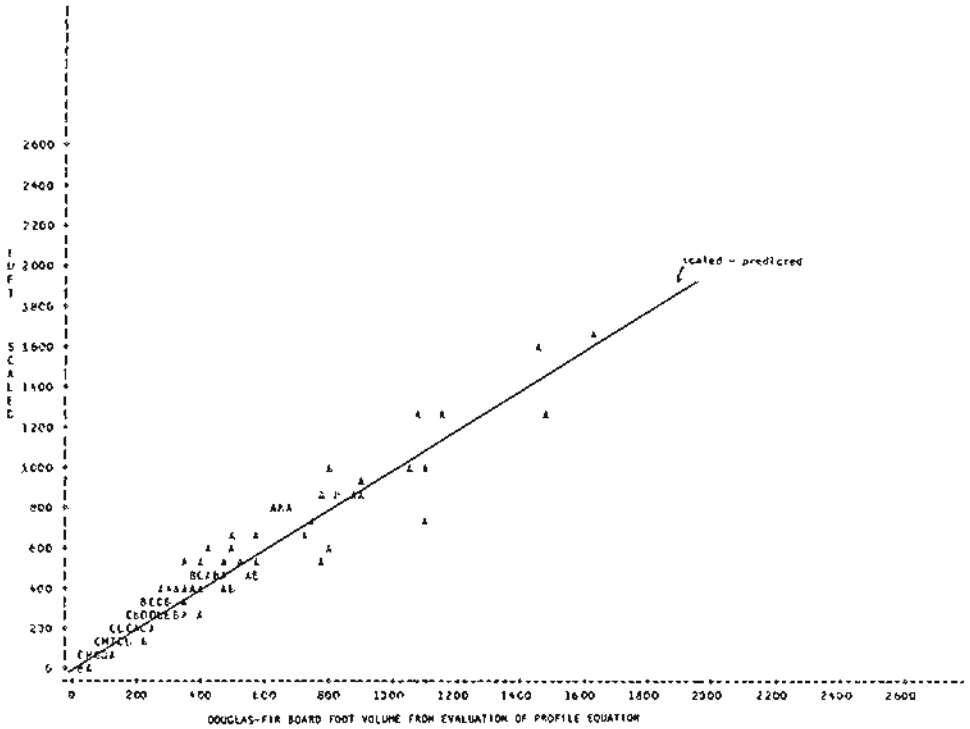

FIGURE 18. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR DOUGLAS-FIR.

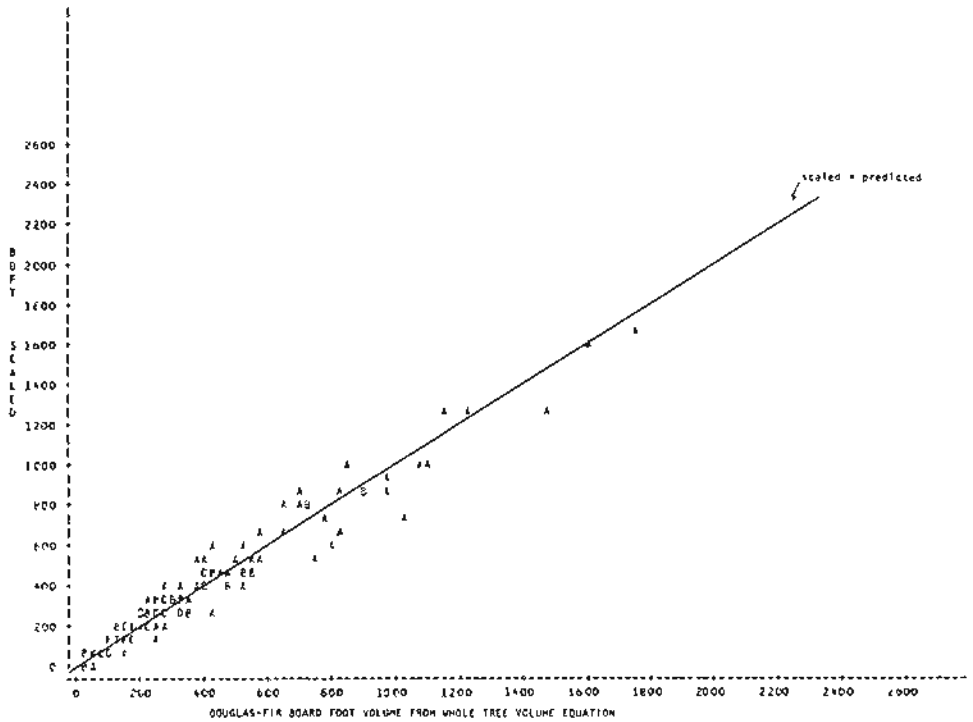

FIGURE 19. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR DOUGLAS-FIR.

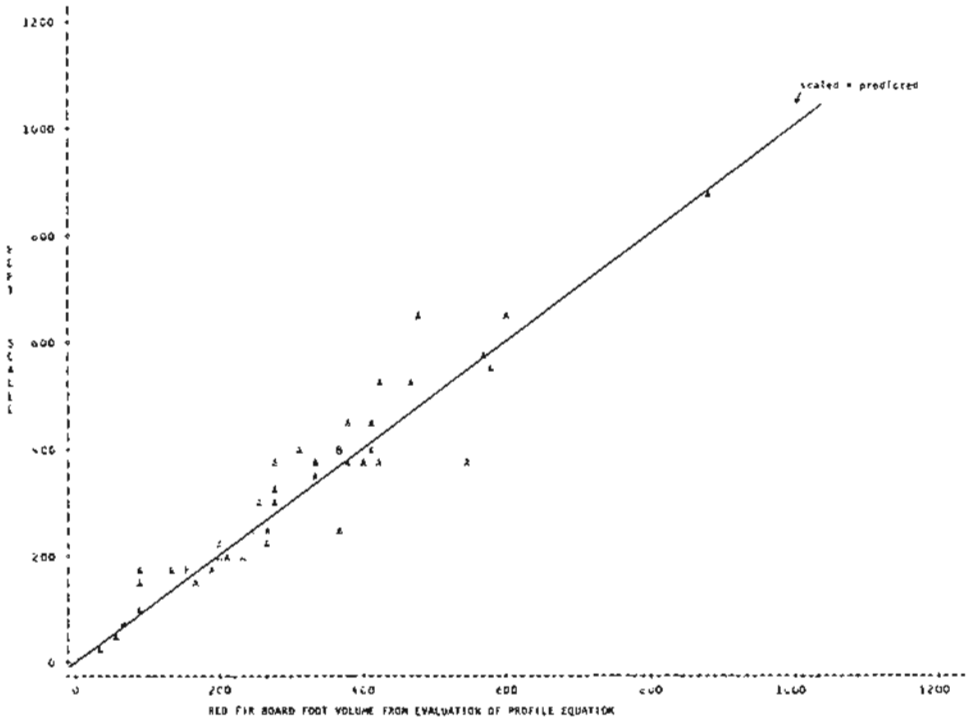

FIGURE 20. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR RED FIR.

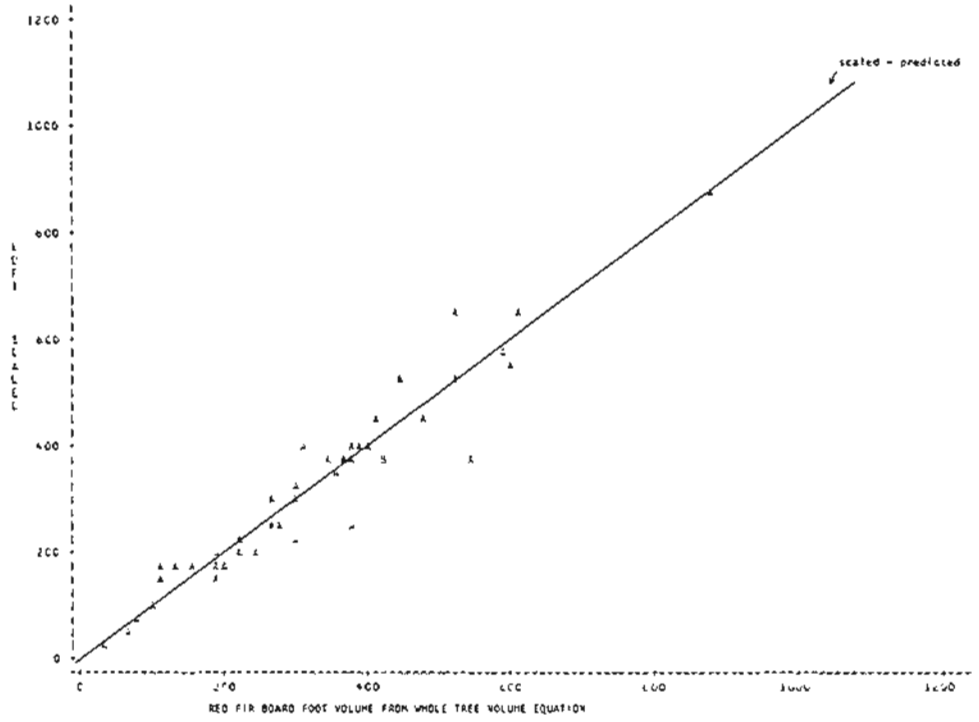

FIGURE 21. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR RED FIR.

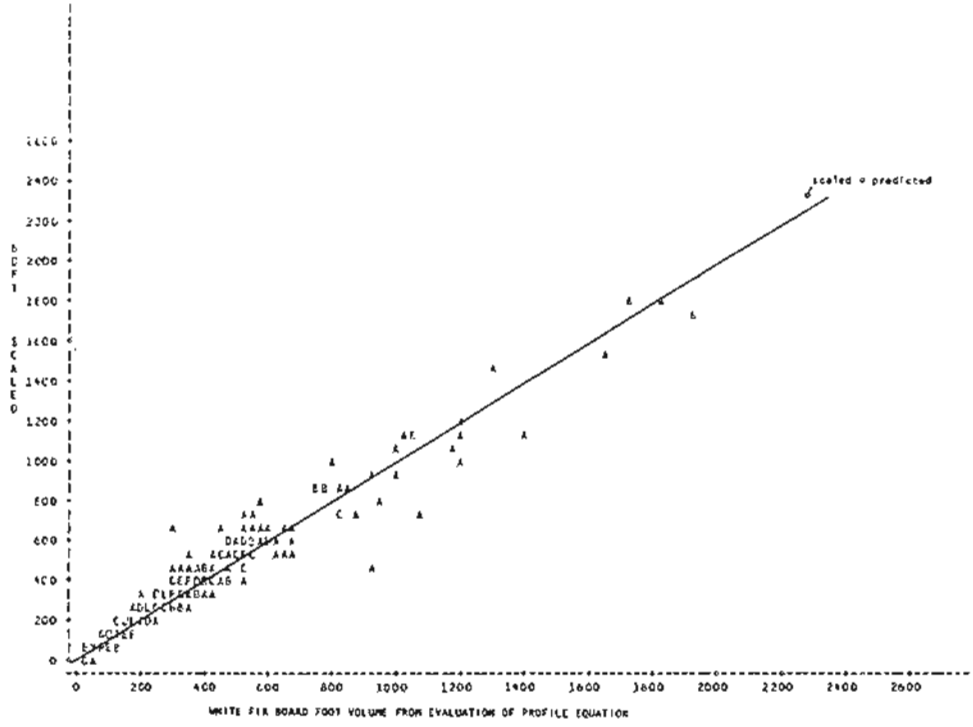

FIGURE 22. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR WHITE FIR.

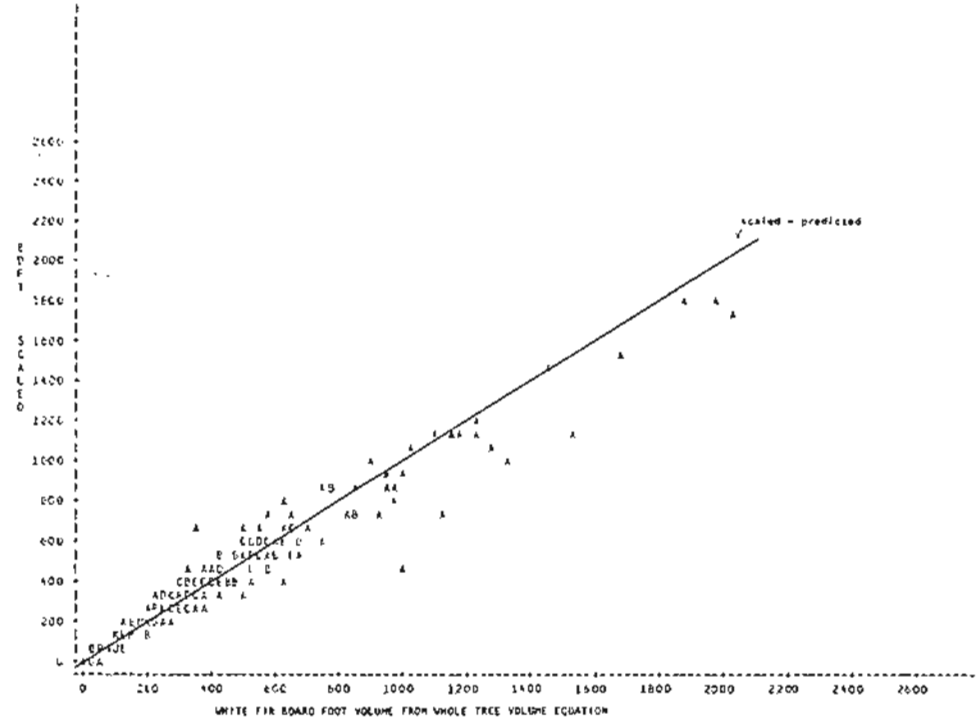

FIGURE 23. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR WHITE FIR.

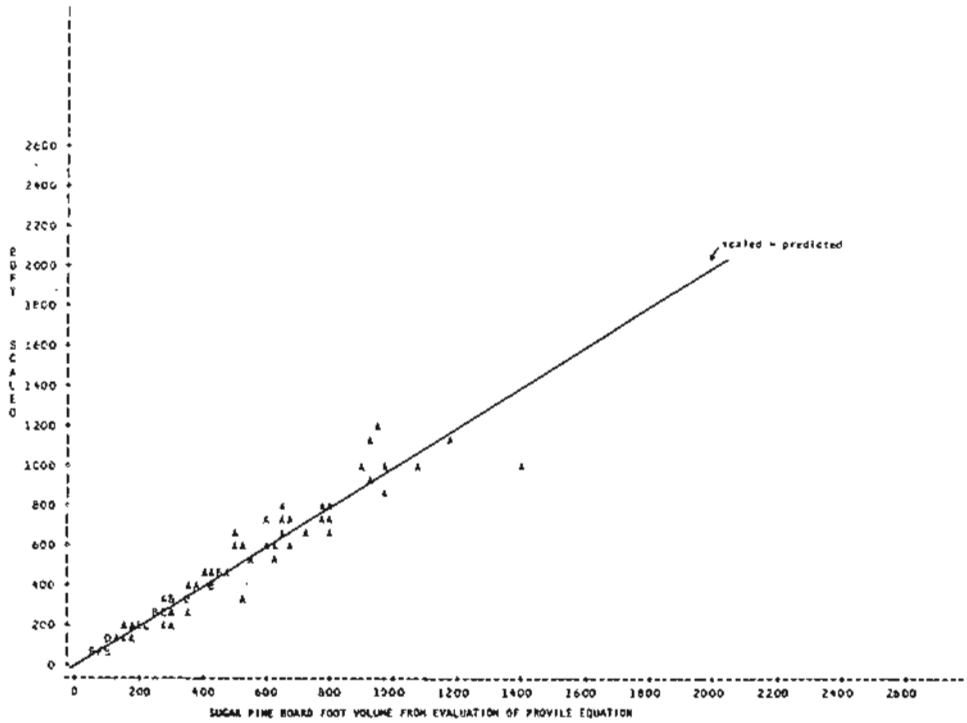

FIGURE 24. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR SUGAR PINE.

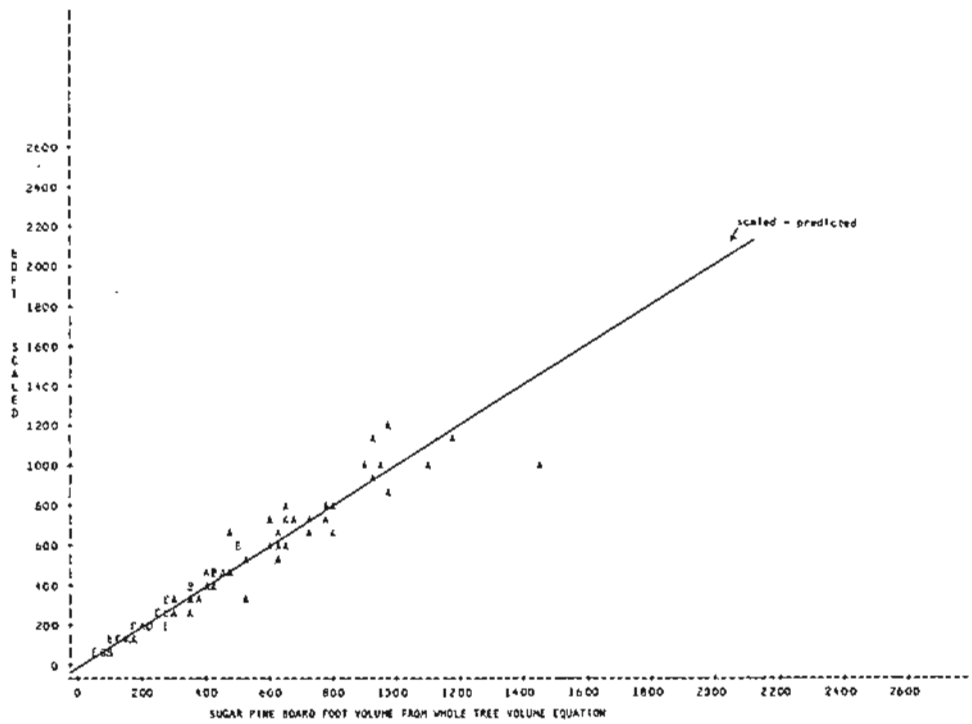

FIGURE 25. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR SUGAR PINE.

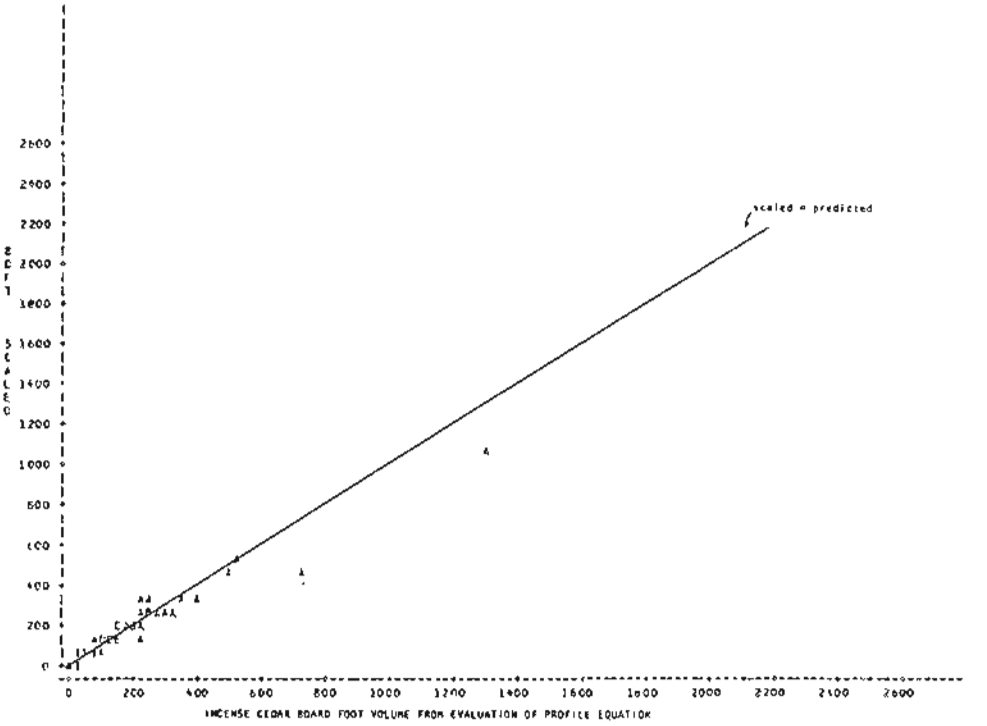

FIGURE 26. PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR INCENSE CEDAR.

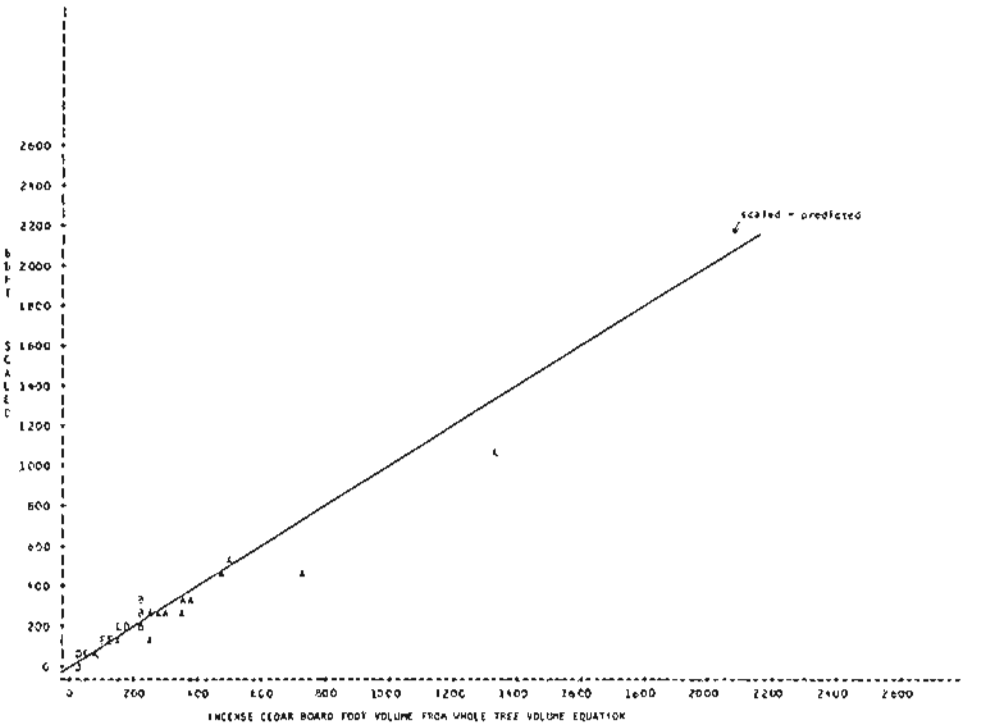

FIGURE 27 PREDICTED BOARD FOOT VOLUME VERSUS SCALED FOR INCENSE CEDAR.

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APPENDIX A.

The integral form of equation (I) for use in direct evaluation of cubic foot contents to any height.

Let D = DBH
H = total height
b = height to top point in the upper stem.

a = height to some base point (not necessarily the stump)

h = height to some point in the stem between a and b.

$\lambda = 1 - \exp(-b_1/b_2)$

$q = 1 - \lambda(h/H)^{1/3}$

$K_1 = .00545\ b_1^2 DBH^2$

$K_2 = .00545\ 2b_1 b_2 DBH^2$

$K_3 = .00545\ b_2^2 DBH^2$

$a' = a/H$

$b' = b/H$

$a'' = 1 - \lambda(a')^{1/3}$

$b'' = 1 - \lambda(b')^{1/3}$

$d = DBH[b_1 + b_2 \ln(1 - \lambda(h/H)^{1/3})]$

Then volume is given as:

$$V = \int_a^b .00545\ d^2(h)\,dh$$

$$= K_1 H\ (b'-a')$$

$$+ K_2 H[-\frac{3}{\lambda^3}]\ [q \ln(q) - q - q^2 \ln(q) + \frac{q^2}{2} + \frac{q^3}{3} \ln(q) - \frac{q^3}{9}]\ \Big|_{a''}^{b''}$$

$$+ K_3 H[-\frac{3}{\lambda^3}]\ [\frac{q^3 \ln^2(q)}{3} - q^2 \ln^2(q) + q \ln^2(q) - \frac{2}{9} q^3 \ln(q)$$

$$+ q^2 \ln(q) - 2q \ln(q) + \frac{2}{27} q^3 - \frac{q^2}{2} + 2q]\ \Big|_{a''}^{b''}$$