Estimation of crown form for six conifer species of northern California

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Geometric models are presented for the prediction of crown volume and width at any height in the crown of six conifer species in the Sierra Nevada. Crown volume is defined as the geometric space occupied by the crown and is allometrically related to the diameter, height, and crown ratio of individual trees. Crown diameter is derived from crown volume, tree height, and crown ratio. The crown volumes and associated measures can be used to compute indices of individual tree competition such as those used in the CACTOS (California Conifer Timber Output Simulator) system or to compute other measures such as wildlife habitat suitability or insect damage potential. Estimation equations are developed by regression using data collected on crowns of 593 felled trees. The equations use dbh, total height, and crown ratio to estimate total crown volume, crown volume above a specified height, and cumulative crown cross sectional area at a specified height.

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Cet article propose des modèles géométriques pour prédire la largeur et le volume de la cime à n'importe quelle hauteur chez six espèces de conifère de la Sierra Nevada. Le volume de cime est défini comme l'espace géométrique occupé par la cime et est relié de façon allométrique au diamètre, à la hauteur et au rapport hauteur de cime – hauteur totale de chaque arbre. Le diamètre de cime est obtenu à partir du volume de cime, de la hauteur et du rapport hauteur de cime – hauteur totale. Les volumes de cime et les mesures qui y sont reliées peuvent être utilisés pour calculer des indices de compétition pour chaque arbre comme ceux qui sont utilisés dans le système CACTOS («California Conifer Timber Output Simulator»), ou pour calculer d'autres variables comme la qualité de l'habitat pour la faune ou les dommages potentiels par les insectes. Les équations de prédiction ont été développées à partir de données sur la cime de 593 arbres abattus. Les équations utilisent le dhp, la hauteur total et le rapport hauteur de cime – hauteur totale pour estimer le volume total de cime, le volume de cime au-dessus d'une hauteur donnée et la surface cumulative d'une section transversale de la cime à une hauteur donnée.

[Traduit par la revue]

Introduction

Nonlinear regression models that estimate tree crown taper and geometric volume have a number of uses, including development of competition measures. Moeur (1981), for example, discusses the application of crown width models developed in the Intermountain Region of the USDA Forest Service for predicting wildlife habitat and for predicting watershed response to land-use changes. Using preliminary results from our research, Morrison et al. (1987) found geometric crown volume in the lower canopy to be an important predictor of bird abundance in the Sierra Nevada. Mawson et al. (1976) describe using 15 geometric shapes to characterize crown volume for trees and large shrubs that can be used for bird habitat studies. Volumes were determined by knowing critical measurements of the shape, such as the height and radius of a cone. Their method depends upon careful field identification of the profile shape because of its influence on the estimate of volume. Hamilton (1969) conducted a growth study on Sitka spruce (*Picea sitchensis* (Bong.) Carr.). He measured the crown radius of 60 standing trees and calculated conical crown volume and surface area, crown projection area, and crown depth from these measurements. He found that crown projection area and crown surface area were important predictors in volume crement.

Growth and yield studies (cf., e.g., Daniels and Burkhart 1975; Ek and Monserud 1974; Krumland and Wensel 1981; and Wensel et al. 1987) have employed a paradigm of growth that has two major components: (*i*) potential growth and (*ii*) reduction due to competition. Potential growth is usually defined as growth obtained for open-grown trees. The competition component reflects the reduction in growth when trees compete for scarce resources.

Crown diameter has been the basis of several competition indices involving area of crown overlap of the subject tree with adjacent trees. Gerrard (1969), Bella (1969), and Arney (1973) are among the earlier developers of competition indices based upon crown width. Krumland (1982) and Wensel et al. (1987) found that a competition index based upon predicted crown cross sectional area for the stand evaluated at two-thirds of the subject trees' height provided a logical and effective measure of competition.

Mitchell (1975) used the ratio of predicted foliage volume to maximum foliage volume as a measure of competition, viewing the crown as five concentric layers of live foliage as determined by branch terminal growth. Other than Mitchell's (1975) work, crown volume has rarely been considered for a competition measure, possibly from lack of data.

We define crown volume as the simple geometric space occupied by the crown. For this study a detailed empirical investigation was undertaken to determine actual crown volumes and to fit prediction models to these data. This is in contrast with prior work such as Mawson et al. (1976), in which crown volumes were roughly approximated and which had limited empirical data from which to characterize crown relationships. We develop models to describe the crown volume and the width of the crown at varying heights and other ancillary measures, such as crown surface area, and discuss their application for six conifer species in northern California.

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TABLE 1. Summary statistics for the data used to model crown volumes

	dbh (in.)		Height (ft)		Crown volume (ft^3)		No. of trees	No. of crown radius measurements
	Mean	SD	Mean	SD	Mean	SD		
Ponderosa pine	19.08	6.47	98.56	24.67	7563.31	6248.85	156	1194
Sugar pine	19.74	6.49	89.47	22.33	7388.81	6520.07	58	454
Douglas-fir	15.87	6.05	85.81	24.36	9650.32	8688.29	115	800
White-fir	15.79	5.85	77.94	23.19	4811.36	3893.57	190	1340
Red fir	18.82	4.61	85.09	13.08	4632.94	3152.42	19	118
Incense cedar	13.57	6.26	56.85	20.62	3448.25	4986.13	55	362
All species	16.95	6.40	84.29	26.02	6593.68	6360.01	593	4268

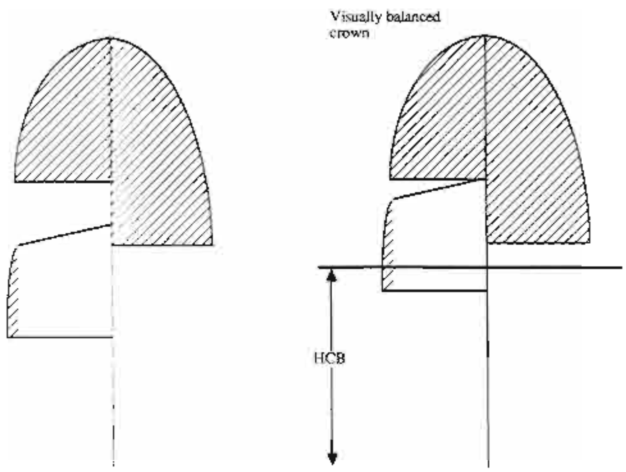

FIG. 1. Determination of height to crown base (HCB) by averaging.

Data sources

Data were collected from 39 plot clusters (109 total plots) in the mixed conifer region of California. Collections were made in cooperation with the forest industry members of the Northern California Forest Yield Cooperative. On each plot, three to six site index trees and up to seven additional trees were felled from the full range of diameters on each plot. The felled trees were bucked beginning at a 1.5-ft stump (1 ft = 0.30 m), breast height (4.5 ft), and thereafter every 16.5 or 20.5 ft.

Because these data were obtained as part of a stem analysis study, our crown measurement points corresponded to the buck points of the tree bole. At the top of each log within the live crown, the left and right crown widths from the stem center were recorded to the nearest foot. Crown width was measured as the trees lay horizontally on the ground. Logs were cut regardless of merchantability until approximately 15 ft from the tree tip. Then three additional crown measurements were taken at 5-, 10-, and 15-year-old branch whorls to represent the crown profile in the tree tip. Other tree data included dbh, total height, and both the Dunning and Keene crown classes (Daniel et al. 1979). Table 1 gives the basic statistics on the major variables of interest for trees used in this study.

A total of 1039 stem analysis trees were candidates for inclusion in this analysis. The data were screened for incomplete records, unacceptably long interpolations or extrapolations, and other inconsistencies. This left 593 trees that could be used reliably in analysis.

Methods

Crown volume determination

Graphical inspection of the data supported the concept of quadratic crown taper. Mawson et al. (1976) also found conifer crown profiles to be mainly parabolic. After screening the data, the sectional volume and total geometric cubic volume of crowns were computed using an interpolation and extrapolation algorithm.

The height to base of the live crown was determined when the tree was still standing so that this measure could be used as a regressor in the crown taper and volume models. Typically, the bottom of the crown was visually averaged (balanced) to find the point that represents the average height to crown base (see Fig. 1). Because of this averaging process, and because crown radii measurements cannot be obained from all sides of the tree after felling, the bottom crown radius measurements taken after the trees were fell could be at heights either above or below the ocularly averaged height to crown base measured on standing trees. If the bottom radii were below height to crown base then an interpolation routine was used to estimate the radii at the crown base. If the bottom radii were above the height to crown base, then an extrapolation routine was used to estimate the radii at the crown base. In both cases, interpolation or extrapolation was limited to less than 20 ft.

Base and intermediate crown sectional volumes were computed with Smalian's formula (Husch et al. 1982). The volume of the first section above the crown base was calculated using the estimated (interpolated or extrapolated) radii associated with the ocularly estimated crown base and the next higher measurements of crown radius. The sectional volume was the product of the quadratic mean of the two radii at the top of the section, the two radii at the bottom of the section, and the height of the section times π. The tip volume was computed assuming a conical model. To ensure the conical model was tenable, tip sections needed to be under 20 ft in length. Total crown volume for each tree was determined by summing the sectional volumes.

This work has some analogies to conventional tree volume and taper work. However, there are notable differences. Tree bole volume is symmetric with respect to the vertical axis of the tree. This is not always true for crown profile since a tree crown may expand in a segment, say a hemisphere, to take advantage of openings in the canop Crown measurements such as height to crown base a. radius at a given height may represent averaged values. Thus crown measurements are taken with different rules and standards than bole values. Tree taper equations exploit the knowledge of dbh to improve the accuracy of the prediction. There is no corresponding analogy to crowns since most inventories do not include, say, measurements of crown width at the crown base. In our methods, we predict crown volumes based upon regressions developed from felled trees using the easily measured variables dbh, height, and live-crown ratio. From the prediction of crown volume we then derive estimates of tree taper and surface area. We initially tried to predict the crown width at the base of the live crown and to develop a taper equation as a function of height above the crown base. We abandoned developing a formal crown taper model because measurements of crown width or radius were not always monotonically decreasing from the crown base to the tip of the tree. Model forms that predicted relative crown width as a function of relative height above the crown base could not be used because of this data limitation. Because of these reasons, we opted to develop a geometric crown volume model and to derive taper and crown surface area from it.

Total geometric crown volume

Total measured cubic foot crown volumes (CV) were highly correlated with dbh, total height (H), and crown ratio (CR). The predicted total cubic foot crown volume ($\widehat{CV}$) was expressed as

$$[1] \quad \widehat{CV} = a(\mathrm{dbh}^b)H^c\mathrm{CR}^d$$

where

$$\mathrm{CR} = \frac{H - \mathrm{HCB}}{H}$$

and a, b, c, and d are coefficients estimated by regression techniques, and HCB is height to crown base.

Equation 1 can be viewed as a function that predicts the maximum potential crown volume, $\widehat{CV}$, for a tree of given dimensions. It is nonlinearly modified by crown ratio and gives a zero volume when the crown ratio, CR, is zero. The logarithmic transform of eq. 1 was fit with ordinary least squares methods as discussed in the Results section.

Cumulative geometric crown volume

A second equation was developed to predict cumulative geometric crown volume at any height in the crown. The equation selected was an adaptation of a cumulative cubic volume function for the stem volume of sweetgum trees (*Liquidambar styraciflua* L.) developed by Van Deusen et al. (1982).

Given the predicted total crown volume, $\widehat{CV}$, the volume above height h is given by

$$[2] \quad V(h) = \widehat{CV}\left(\frac{H - h}{H - \mathrm{HCB}}\right)^k, \qquad \text{for HCB} \le h \le H$$

where

$V(h)$ = the cumulative crown geometric volume (ft^3) from the crown base to a height h

$\widehat{CV}$ = total predicted geometric cubic volume (ft^3) defined as in [1]

HCB = height to live crown base (ft)

H = total tree height (ft)

k = a species-specific parameter that determines the shape of the profile

Equation 2 is constrained to predict zero cumulative volume at total tree height ($h = H$) and to predict full geometric crown volume at crown base (h = HCB).

Crown cross sectional area equation

Let CA(z) denote crown cross sectional area (ft^2) at height z on the interval HCB $\le z \le H$. Then a cross sectional area expression can be developed by expressing the crown volume above height h as the integral of the crown area from the tip to the base of the crown as follows:

$$[3] \quad V(h) = \int_H^h \mathrm{CA}(z)\mathrm{d}z$$

Setting eqs. 2 and 3 equal we obtain

$$\widehat{CV}\left(\frac{H - h}{H - \mathrm{HCB}}\right)^k = \int_H^h \mathrm{CA}(z)\mathrm{d}z$$

After taking the derivative of both sides of the equation we get

$$\frac{-k\,\widehat{CV}}{H - \mathrm{HCB}}\left(\frac{H - h}{H - \mathrm{HCB}}\right)^{k-1} = \mathrm{CA}(h)$$

Because we normally would be evaluating h on the interval HCB $\le h \le H$, we need to invert the order of integration from (H,h) to (h,H). It follows that

$$[4] \quad \mathrm{CA}(h) = \frac{k\,\widehat{CV}}{(H - \mathrm{HCB})}\left(\frac{H - h}{H - \mathrm{HCB}}\right)^{k-1} \qquad \text{for HCB} \le h \le H$$

Equation 4 can also be rearranged to produce an alternative means of expressing total crown volume. In this prediction, crown volume is a function of crown cross sectional area CA(h) at height h, giving

$$[5] \quad \widehat{CV}' = \frac{\mathrm{CA}(h)}{k}(H - \mathrm{HCB})\left(\frac{H - h}{H - \mathrm{HCB}}\right)^{-(k-1)} \qquad \text{for HCB} \le h \le H$$

This equation provides another estimate of total geometric crown volume when information on crown diameter (or cross sectional area) and k are available. The prime indicates that crown volume is estimated in an alternative manner than in eq. 1. However, eq. 5 should only be used in specific cases as discussed in a following section.

The following section derives relationships from the equations listed in the previous section.

Crown surface area

A model for cumulative crown surface area can be defined using the folowing formula for a surface of revolution[1] (Thomas 1966; Husch et al. 1982):

$$[6] \quad \mathrm{CSA}(h) \approx 2\pi \int_h^H r(z)\mathrm{d}z$$

[1]There are two potential approximations to surface area. The first is $\Delta S \approx 2\pi r\Delta h$, and the second is $\Delta S \approx 2\pi r\sqrt{\Delta h^2 + \Delta r^2}$. We use the first approximation because of the simplicity of the solution and since only an index to surface area is required. However, the second approximation is more strictly correct. For our data the average ratio of crown radius (at the crown base) to crown length was 0.2. Using $\Delta r/\Delta h$ as an estimate of the derivative, it follows that there would be approximately a 2% error in estimation. Over 80% of the trees had $\Delta r/\Delta h$ ratios that were less than 0.3. Thus we would expect that the majority of trees would have less than a 4.5% error in estimation of surface area using this approximation technique.

TABLE 2. Parameter values and fit statistics for eq. 9

	Pines[a]	Douglas-fir	Incense cedar	White fir	Red fir[b]	All species
a	4.819 46	14.552 86	2.236 20	11.069 46	9.571 808	7.911 89
$\hat{a}^c$	5.286 90	16.235 61	2.494 70	11.983 69	9.571 808	8.909 03
b	1.314 45	0.976 45	1.549 16	0.951 71	0.951 71	1.124 50
d	1.921 75	1.462 73	1.997 50	1.564 05	1.564 05	1.727 22
$S_{y.x}(\ln(\mathrm{ft}^3))$	0.430 29	0.467 80	0.467 74	0.398 38	0.327 17	0.487 23
$S_{y.x}(\mathrm{ft}^3)$	3295.5	4485.0	1870.1	1922.7	1391.2	3527.3
No. of observations	214	115	55	190	19	593

NOTE: Let $y = [\ln(\mathrm{CV}) - \ln(H)]$, then $\hat{y} = \ln(a) + b\ln(\mathrm{dbh}) + d\ln(\mathrm{CR})$.
[a]Ponderosa pine and sugar pine.
[b]Because of the infrequency of the red fir observations, the red fir coefficients were fitted using the white fir model form with the b and d coefficients held the same as for white fir. However, the a coefficient was allowed to vary.
[c]The estimate of the a coefficient after conversion from the logarithmic estimate and corrections for log bias. We use $S^2_{y.x}/2$ as the correction factor to predict the mean value rather than the median value. Volume is estimated as $V = \hat{a}\,D^b\,H\,\mathrm{CR}^d$, where $\hat{a}$ is defined as exp(ln(a) + log bias correction factor). For red fir, the a coefficient was estimated as a ratio (observed/(D^bH CR^d)), and thus, no log bias correction was required.

where

CSA(h) = cumulative crown surface area (ft^2) to any height (h) above ground with HCB $\le h \le H$

$r(z)$ = radius of the crown at height z for HCB $\le z \le H$

By noting that $\mathrm{CA}(z) = \pi r^2(z)$ and by using a change of variables technique, it can be shown that

$$\mathrm{CSA}(h) \approx 2\pi \int_h^H \sqrt{\left(\frac{\mathrm{CA}(z)}{\pi}\right)}\,dz$$

or, from eq. 4

$$[7] \quad \mathrm{CSA}(h) \approx 2\pi \int_h^H \sqrt{\frac{k\,\widehat{\mathrm{CV}}}{\pi\,(H - \mathrm{HCB})}\left(\frac{H - z}{H - \mathrm{HCB}}\right)^{k-1}}\,dz$$

After integration and algebraic manipulation of eq. 7 we get

$$[8] \quad \mathrm{CSA}(h) \approx \frac{4}{k+1}\sqrt{\pi k\,\widehat{\mathrm{CV}}(H - \mathrm{HCB})} \times \sqrt{\left(\frac{H - h}{H - \mathrm{HCB}}\right)^{k+1}}$$

for h in the range HCB $\le h \le H$.

As an alternative to this procedure, actual crown surface area can be determined in a similar manner to that used in determining actual crown volume as discussed in the beginning of the Methods section. A separate crown surface area predictive equation similar in form to eq. 2 could then be developed. However, such a predictive equation would then be incompatible with eq. 8.

Results and applications

In the following section, the results for the crown volume, crown cross sectional area, crown surface area, and cumulative crown volume models are presented.

Geometric crown models

The crown volume predictive model [1] is a nonlinear function with four parameters to be estimated. Using nonlinear least squares,[2] the value of the c parameter for the individual species ranged from 0.5 to 2.3. Considering all species combined, the c parameter was highly correlated with parameters a and b ($r = -0.9$ and -0.8, respectively). Because of this high correlation, the c value was fixed to a central value of 1.0 and a, b, and d were reestimated. This had a minimal effect on the residual sum of squares and mean square error, increasing them by only approximately 1.5% compared with the full model.

A natural logarithmic transformation (denoted ln) was used on model [1] so that the coefficients could be estimated with ordinary least squares and to more nearly meet the assumption of homogeneity of variance. The transformed model can be written as

$$[9] \quad \ln(\widehat{\mathrm{CV}}) = \ln(a) + b\ln(\mathrm{dbh}) + c\ln(\mathrm{H}) + d\ln(\mathrm{CR}) + e$$

where c, as noted earlier, was specified to be 1.0.

Coefficients and fit statistics are presented in Table 2 for model [9]. In general, red fir (*Abies magnifica* A. Murr.), white fir (*A. concolor* (Gord. & Glend.) Lindl.), and incense cedar (*Libocedrus decurrens* Torr.) had superior model fits in comparison to ponderosa pine (*Pinus ponderosa* Laws.), sugar pine (*P. lambertiana* Dougl.), and Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco). An analysis of covariance was conducted to see if the models for ponderosa pine and sugar pine could be combined into a "pines" model and whether the red and white fir models could be combined into a "fir" model. At a α level of 0.05, the hypothesis that the pine models were equivalent could not be rejected. The hypothesis that the fir species could be combined was not accepted. Because of the paucity of red fir observations, however, only the a coefficient was fitted to the data, with the b and d coefficients being held the same as for white fir.

Cumulative geometric crown volume

The cumulative crown volume model is a nonlinear function with one parameter to be estimated. Coefficients and fit statistics are presented in Table 3. In general, the firs (red and white) and incense cedar had regression standard

[2]SAS nonlinear regression routines (see SAS Institute Inc. 1985) were employed using Marquardt's compromise for locating the minimum residual sum of squares.

TABLE 3. Parameter values and fit statistics for models [2], [4], and [5]

	k estimated from:			Sample size (n)		
	model [2]	model [4]	model [5]	Model [2]	Model [4][a]	Model [5]
Pines[b]	1.923 73	1.784 78	2.384 405	824	821	128
Douglas-fir	2.034 07	1.805 02	2.647 413	400	400	62
Incense cedar	1.895 09	1.702 38	2.455 147	181	179	34
White fir	2.176 27	2.004 06	2.524 078	670	665	97
Red fir	2.236 84	2.040 63	2.515 075	59	59	7
All species	1.999 55	1.831 80	2.520 392	2133	2124	328

	SE (ft^3) for model [2] with k estimated from:		SE (ft^2) for model [4] with k estimated from:		SE (ft^3) for model [5] with k estimated from:	
	model [2]	model [4]	model [2]	model [4]	model [4]	model [5]
Pines[b]	698	725	77.7	73.6	11 203	2387
Douglas-fir	738	856	97.0	87.8	17 331	2904
Incense cedar	306	355	56.0	52.1	7 981	2642
White fir	292	319	65.8	44.4	7 057	1064
Red fir	201	254	73.2	34.6	7 152	1220
All species	580	620	69.5	66.5	11 304	2213

[a]The number of data points is slightly less than for model [2] since cross sectional areas less than 3 ft (crown radius < 1 ft) were not used in estimating the cross sectional profile relationships.
[b]Ponderosa pine and sugar pine.

errors that were less than one-half the value of the standard errors for pines (ponderosa and sugar) and Douglas-fir. This result was very similar to that reported for the total geometric crown volumes in model [9].

The k-parameter in the cumulative geometric crown volume model [2] also appears in eq. 4 for crown cross sectional area (see following section). Thus, this parameter can be estimated from either model. Use of eq. 4 to estimate k would imply that more accuracy is desired in estimating crown cross sectional area than in estimating cumulative crown volumes. Since a major application of the models developed herein lies in computing a competition index for an individual tree growth and yield simulator (Wensel et al. 1987) based on crown cross sectional area, model [4] was the preferred model for which to estimate the k-parameter for our applications. Nonetheless, the ability of the coefficient k, estimated from the cross sectional area model [4], to characterize the cumulative geometric crown volume relationship represented by model [2] is investigated. Table 3 provides the fit statistics computed using model [4] coefficients substituted into model [2]. There is approximately a 7–11% decrease in the estimate of the k-parameter and an increase in the standard error ranging from 4 to 25%, but averaging 7%. Thus, there is some loss in precision from using the k-parameter estimate from the cross sectional model to estimate cumulative geometric crown volume.

Crown cross sectional area equation

Estimates of k and fit statistics are presented in Table 3 for model [4]. As for eqs. [9] and [2], models for the firs and incense cedar fit better than models for the pines and Douglas-fir. Since the firs are shade tolerant and grow in relatively dense stands, we may be observing a density effect for these species that may result in more uniform crowns.

As stated in the prior section, the k-parameter in the cumulative geometric crown volume model [2] also appears in eq. 4 for crown cross sectional area. Hence, the model fit statistic ($S^2_{y.x}$) computed using model [2] coefficients substituted into model [4] is presented in Table 3. In the second case, the model fit reported represents the ability of cumulative crown volume coefficients to represent the cross sectional area relationships. The use of model [2] coefficients provides relatively accurate fits for the cross sectional area in all species but red and white fir. Because of the preceding arguments, estimates of the k-parameter fitted to model [4] are preferred.

Derived total geometric crown volume given a specific crown cross sectional area

Equation 5 provides an estimate of crown volume when information on height, height to crown base, and crown diameter or cross sectional area is available. The accuracy of this estimate of crown volume depends upon the relative height ($(H - h)/(H - \mathrm{HCB})$) at which cross sectional area is measured. Nonlinear regression was used to estimate k for all species combined for the following relative height classes: 0–0.2, 0.2–0.4, 0.4–0.6, 0.6–0.8, and 0.8–1.0. Since mean squared errors were substantially lower for the 0.6–0.8 relative height class, k was estimated independently for each species using data from this class (Table 3).

It appears that knowing cross sectional area at some point in the relative height range of 0.6–0.8 improves the overall estimate of total crown geometric volume for all species except incense cedar in comparison with model fits for total volume presented in Table 2. This prediction method may prove superior, but it cannot be fully judged by comparing the two estimates of residual variance because the values presented in Table 3 are assessed only for a total of 328 data points in the relative height range of 0.60 to 0.80, whereas model [9] coefficients were developed based upon total volume estimated for 593 trees. Thus, some caution should be exercised in interpreting this to mean that there has been a large reduction in residual sum of squares error for estimating total crown volume. Nonetheless, it appears that this method can improve the total crown volume estimate when crown cross sectional area can be accurately measured in the range of relative height between 0.6 and 0.8. We expect, however, that it would be difficult to make these measurements accurately as part of a field inventory procedure.

Conclusions

The models developed herein to describe crown diameter, volume, and surface area have many applications, including development of individual tree competition measures, wildlife habitat suitability assessment, and calculating insect damage potential. These equations have been successfully applied to the first two areas (cf. Wensel et al. 1987; Morrison et al. 1987, respectively).

The results of this work demonstrate that with relatively simple models (one to four parameters) it is possible to characterize the crown geometry (total geometric crown volume, cumulative crown volume, and crown cross sectional area) for the six major mixed conifer species of northern California. Two alternative methods for estimating the *k*-coefficient common to the crown cross sectional area and cumulative crown volume models (models [4] and [2], respectively) were investigated. The regression coefficients estimated from each equation were evaluated in their ability to fit the data using the second model form. The *k*-coefficients could be estimated by fitting either the crown cross sectional or the cumulative crown volume models and then used to provide relatively precise predictions for the other variate. Because the primary end use of this work by the authors was to develop crown cross sectional area equations for tree competition indices, the coefficients estimated with eq. 4 are preferred.[3] Using these equations and applying principles of calculus, it was also possible to derive a crown surface area equation that was compatible with the prediction of crown volume and taper.

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[3]For other applications it may be desirable to simultaneously minimize the sum of volume and cross sectional area deviations squared.

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